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φ-means of some Banach subspaces on a Banach algebra

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Abstract

In this paper, among the other things, we study the concept of φ-amenability of a Banach algebra A, where φ is a nonzero multiplicative linear functional on A. We present a few results in the theory of φ-amenable Banach algebras, and we obtain necessary and sufficient conditions for A^{**} to have a left invariant φ-mean on Banach subspaces of A^*. The candidates for the choice of space are A_*, $WAP(A)$ and $S(G)$.

Keywords: Banach algebra, φ-amenability, φ-means, weak* topology.
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1 Introduction

In [3], Lau introduced and investigated a large class of Banach algebras which he called F-algebras. Later, F-algebras were termed Lau algebras. They are Banach algebras A such that the dual A^* is a von Neumann algebra and the identity of A^* is a multiplicative linear functional on A. The concept of left amenability for a Lau algebra has been extensively extended for an arbitrary Banach algebra by introducing the notion of φ-amenability (see [2]). Let A be an arbitrary Banach algebra and φ a character of A, that is a homomorphism from A onto $\mathbb{C}$. A is called φ-amenable if there exists a bounded linear functional m on A^* satisfying $\langle m, \varphi\rangle = 1$ and $\langle m, f.a\rangle = \varphi(a)\langle m, f\rangle$ for all $a \in A$ and $f \in A^*$. This concept considerably generalizes the notion of left amenability for Lau algebras.

The main purpose of this paper is to investigate the φ-amenability for certain Banach subspaces of dual Banach algebras. We continue [1] in the study of amenability of a Banach algebra A defined with respect to a character φ of A. Various necessary and sufficient conditions are found for a Banach algebra to possess a left invariant φ-mean. Throughout the paper, $\Delta(A)$ will denote the set of all homomorphisms from A onto $\mathbb{C}$.

We prove that A^{**} has a left invariant φ-mean on A_* if and only if for every normal φ-bimodule E, every bounded weak*-continuous derivation $D : A \to E$ is inner. Other results in this direction are also obtained. Our second purpose in this paper is to present several characterizations of the existence of a left (right) invariant φ-mean on $Wap(A)$. Finally we obtain sufficient conditions and some necessary conditions about $S(G)$ to have a left invariant 1-mean.

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