Annihilator Conditions in Noncommutative Ring Extensions

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Abstract

Let R be a ring, S a strictly ordered monoid and $\omega : S \to \mathrm{End}(R)$ a monoid homomorphism. In [4], Marks, Mazurek and Ziembowski study the class of (S,ω)-Armendariz rings, as a generalization of the standard Armendariz condition from ordinary polynomial ring to skew generalized power series ring. We observe from results in [4], that the upper nilradical coincides with the prime radical in (S,ω)-Armendariz rings and also every one-sided nil ideal of these rings is contained in a two-sided nil ideal of the ring, namely satisfies in the Köthe's conjecture. Also it can be shown that the factor rings of an (S,ω)-Armendariz rings over its prime radical is also (S,ω)-Armendariz. We continue in this paper the study of rings with such property in skew generalized power series rings and bring some properties of these rings.

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1 Introduction

Throughout the present paper all rings considered, unless otherwise noted, shall be assumed to be associative and possess an identity; subrings of a ring need not have the same unit, *subrng* will denote a subring without unit, and "an order" on a set will always mean "a partial order". Our notation and terminology are standard and shall follow [3]. For instance, for such a ring R, the monoid of endomorphisms of R (with composition of endomorphisms as the operation) is denoted by $\mathrm{End}(R)$. We adopt the notations $Ni\ell(R)$, $Ni\ell_*(R)$ and $Ni\ell^*(R)$ to represent the set of all nilpotent elements, the lower nilradical (i.e., the prime radical) and the upper nilradical (i.e., the sum of all nil ideals) of a ring R, respectively. By $R[S]$, we mean the monoid ring of a monoid S over a ring R, while $R[x]$ denotes the ring of all polynomials over a ring R.

Let $(S,\leq)$ be an ordered set. Then $(S,\leq)$ is called *artinian* if every strictly decreasing sequence of elements of S is finite and $(S,\leq)$ is called *narrow* if every subset of pairwise order-incomparable elements of S is finite. An *ordered monoid* is a pair $(S,\leq)$ consisting of a monoid S (written multiplicatively) and an order $\leq$ on S such that for all $s_1, s_2, t \in S$, $s_1 \leq s_2$ implies $s_1 t \leq s_2 t$ and $t s_1 \leq t s_2$. An ordered monoid $(S,\leq)$ is said to be *strictly*

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