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Best approximation in normed left modules

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Abstract

We introduce a generalized notion of best approximation and also investigate some basic properties of this notion. Some illustrative examples are presented.

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1 Introduction

Suppose that Y is a normed vector space and K is a non-empty subset of Y. An element $k_0 \in K$ is said to be a best approximation for $y \in Y$, if

$$\|y - k_0\| = d(y, K) = inf\{ \|y - k\| \mid k \in K \}.$$

The set of all best approximations of y in K is denoted by $P_K(y)$. One can easily cheque that if K is closed, then so is $P_K(y)$. The non-empty subset K of Y is said to be proximinal if $P_K(y) \neq \emptyset$ for all $y \in Y$. Also K is said to be Chebyshev, if each point $y \in Y$ has a unique best approximation in K. For the basic results concerning the theory of best approximation, the reader can refer to [1, 3].

Our purpose in this paper is to introduce the module best approximation of the elements of a normed left module and also its module proximinal and module Chebyshev subsets. Also we prove some basic results concerning module best approximation.

For this end we introduce some terminologies. Let A be a non-zero normed algebra, X be a normed left $A-$module and W be a non-empty subset of X. For an element $x \in X$, we say that an element $w_0 \in W$ is an $A-$best approximation for x, if there exists an element $0 \neq a \in A$ such that $ax = x$ and $\|x - aw_0\| = d(x, aW)$. We denote by $(AP)_W(x)$, the set of all $A-$best approximations of $x \in X$ in W. Also we say that W is $A-$proximinal if $(AP)_W(x) \neq \emptyset$ for all $x \in X$, and it is $A-$Chebyshev if each point $x \in X$ has a unique $A-$best approximation in W. The basic properties of the module best approximation in normed left modules are investigated in [2].

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