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Talk On the flag curvature of bi-invariant Randers metrics pp.: 1–4

On the flag curvature of bi-invariant Randers metrics*

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Abstract

In this paper we study the flag curvature of bi-invariant Randers metrics. We first correct a minor error which occurred for the flag curvature formula of a bi-invariant Randers metric. Then we improve this formula on a connected Lie group G and as an application we explicitly give this formula for the Lie groups $SO(4)$ and $U(3)$ which show that these spaces are of non-negative flag curvatures. Some results on the flag curvature formula of a naturally reductive Randers metric are also improved.

Keywords: Flag curvature, Bi-invariant Randers metrics, Connected Lie groups
Mathematics Subject Classification [2010]: 53C60, 53C30.

1 Introduction

The study of invariant structures on Lie groups and homogeneous manifolds is an interesting subject in differential geometry. In the last decade a generalization of these concepts from the Riemannian geometry into the Finsler geometry, specially Randers metrics have been done [1, 2, 3, 4, 5, 6]. One of these invariant structures are bi-invariant metrics and the study of the flag curvature of bi-invariant metrics as a generalization of sectional curvatures in the Riemannian geometry has absorbed a special attention of the mathematics scientists. In particular in [6] an explicit formula for the flag curvature of bi-invariant Randers metrics is given which has a minor error. Our aim in this paper is to correct this formula. We also improve this formula and apply it for calculating the flag curvature of the compact Lie groups $SO(4)$ and $U(3)$. Some interesting results for the flag curvature of naturally reductive are also proved.

2 The flag curvature of a bi-invariant Randers metric

The following formula

$$K(P,y)=\frac{<[y,[u,y]],V>_0 .<V,u>_0+<[y,[u,y]],u>_0 (1+<V,y>_0)}{4(1+<V,y>_0)^2(1-<V,y>_0)}, \qquad (1)$$

is given in [6] for the flag curvature of a Randers metric which is defined by a bi-invariant Riemannian metric g_0 and a left-invariant vector field V which is parallel with respect to g_0. In the correct way it can be written as

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