A note on composition operators between weighted Hilbert spaces of analytic functions

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Abstract

In this paper, we consider composition operators on weighted Hilbert spaces of analytic functions and observe that a formula for the essential norm, give a Hilbert-Schmidt characterization and characterize the membership in Schatten-class for these operators. Also, closed range composition operators are investigated.

Keywords: composition operators, essential norm, Hilbert-Schmidt, Schatten-class, closed range
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1 Introduction

Let $\mathbb{D}$ denotes the open unit disk $\{z \in \mathbb{C} : |z| < 1\}$ and φ be an analytic self map of $\mathbb{D}$. The composition operator C_φ induced by φ is defined $C_\varphi f = f \circ \varphi$, for any $f \in H(\mathbb{D})$, the space of all analytic functions on $\mathbb{D}$. This operator can be generalized to the weighted composition operator uC_φ, $uC_\varphi f(z) = u(z)f(\varphi(z))$, $u \in H(\mathbb{D})$. We consider a *weight* as a positive integrable function $\omega \in C^2[0, 1)$ which is radial, $\omega(z) = \omega(|z|)$. The weighted Hilbert space of analytic functions $\mathcal{H}_\omega$ consists of all analytic functions on $\mathbb{D}$ such that

$$||f'||^2_\omega = \int_{\mathbb{D}} |f'(z)|^2 \omega(z)\ dA(z) < \infty,$$

equipped with the norm $||f||^2_{\mathcal{H}_\omega} = |f(0)|^2 + ||f'||^2_\omega$. Here dA is the normalized area measure on $\mathbb{D}$. Also the weighted Bergman spaces defined by

$$\mathcal{A}^2_\omega = \left\{ f \in H(\mathbb{D}) : ||f||^2_\omega = \int_{\mathbb{D}} |f(z)|^2 \omega(z)\ dA(z) < \infty \right\}.$$

If $f(z) = \sum_{n=0}^\infty a_n z^n$, then $f \in \mathcal{H}_\omega$ if and only if $||f||^2_{\mathcal{H}_\omega} = \sum_{n=0}^\infty |a_n|^2 \omega_n < \infty$, where $\omega_0 = 1$ and for $n \geq 1$

$$\omega_n = 2n^2 \int_0^1 r^{2n-1} \omega(r) dr,$$

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