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On the Zeros of the Elliptic Operator

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Abstract

In this note we discuss about the problem of existence and uniqueness of local extremum points of multi variable zeros of the elliptic operator defined on an open or compact subset of the Euclidean space. We also obtain some results on the theory of partial differential equations.

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1 Preliminaries

Let $U \subseteq R^n$ be an open set, $f : U \to R$ be a map of class C^2 and $\nabla^2 f = \sum_{i=1}^{n} \frac{\partial^2 f}{\partial x_i^2}$. The function f is called Harmonic if $\nabla^2 f = 0$. For the function ε defined on U, the C^2 function $f : U \to R$ is called $\varepsilon-$Subharmonic if $\nabla^2 f = \varepsilon$. For a non empty set $D \subseteq R^n$ the map $f : D \to R$ is called Harmonic on D if there exists an open set U containing D and a map $g : U \to R$ of class C^2 such that $g_{|D} = f$ and $\nabla^2 g = 0$ on D. The set of all Harmonic (res. $\varepsilon-$Subharmonic) functions on D is denoted by $H(D)$ (res. $S(\varepsilon, D)$). Let $A = [a_{ij}]$ be an $n \times n$ positive definite symmetric matrix and $L = (\frac{\partial}{\partial X})A(\frac{\partial}{\partial X})^t$, then L is called an Elliptic operator and the C^2 function $f : U \to R$ is called $L-$Harmonic (res. $\varepsilon L-$Subharmonic) on U if $L(f) = 0$(res. $L(f) = \varepsilon$). The set of all $L-$Harmonic (res. $\varepsilon L-$Subharmonic) functions on U is denoted by $H(L, U)$ (res. $S(\varepsilon, L, U)$). Similarly if $D \subseteq R^n$ be a nonempty set, then the sets $H(L, D)$ and $S(\varepsilon, L, D)$ defined as above.

2 Introduction

The problem of existence of local extremum points of Holomorphic functions is discussed in [1]. Also the similar problem for two variable Harmonic functions defined on a compact subset of the Euclidean plane is proposed in [2] and [5] as the real part of some Holomorphic functions. Dowling [3] showed that an extension of maximum principle for vector valued harmonic functions defined on the open unit disc to a complex Banach space is hold. A new method for finding the extremum points of smooth functions is discussed in [6, 7]. In this note we generalize the similar results for multi variable generalized Harmonic and Subharmonic functions defined on a compact set $D \subseteq R^n$, i.e., the elements f of $H(D)$, $S(\varepsilon, D)$, $H(L, D)$ and $S(\varepsilon, L, D)$ for which L is an elliptic operator defined on $C^2(R^n, R)$. Then we deduce some uniqueness theorems on the theory of Boundary Value Problem $LT = \varepsilon$.

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