Almost one-to-one factor maps on Smale spaces

Sarah Saeidi Gholikandi *
Tarbiat Modares university

Abstract

Almost one-to-one maps between Smale spaces are factor maps whose the degrees are one. Putnam shows that any almost one-to-one factor map between Samle spaces can be decomposed as a composition of two resolving maps. In this paper, we investigate this subject and we show that two resolving maps are almost one-to-one, too.

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1 Introduction

Definition 1.1. [2, 3] Suppose that (X, f) is a compact metric space and f is a homeomorphism of X. Then (X, f) is called a Smale space if there exist constants ε_X and $0 < \lambda < 1$ and a continuous map from

$$\triangle_{\varepsilon_X} = \{(x, y) \in X \times X \quad | \quad d(x, y) \leq \varepsilon_X \quad \}$$

to X (denoted with $[,]$) such that:

$B\,1$ $[x, x] = x$,
$B\,2$ $[x, [y, z]] = [x, z]$,
$B\,3$ $[[x, y], z] = [x, z]$,
$B\,4$ $[f(x), f(y)] = [x, y]$,
$C\,1$ $d(f(x), f(y)) \leq \lambda\, d(x, y)$, whenever $[x, y] = y$,
$C\,2$ $d(f^{-1}(x), f^{-1}(y) \leq \lambda\, d(x, y)$, whenever $[x, y] = x$, whenever both sides of an equation are defined.

Definition 1.2. [2] Two points x and y in X are stably (or unstably) equivalent if

$$\lim_{n\to+\infty} d(f^n(x), f^n(y)) = 0 \qquad (or \lim_{n\to-\infty} d(f^n(x), f^n(y)) = 0, \text{resp.}).$$

Let $X^s(x)$ and $X^u(x)$ denote the stable and unstable equivalence classes of x, respectively.

We recall that a factor map between two Smale spaces (Y, g) and (X, f) is a continuous function $\pi : Y \to X$ such that $\pi \circ g = f \circ \pi$. Of particular importance in this paper are factor maps which are s-bijective: that is, for each y in Y, the restriction of π to $Y^s(y)$ is a bijection to $X^s(\pi(y))$. There is obviously an analogous definition of a u-bijective factor map.[2]

*Speaker