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Some Results concerning 2-frames

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Abstract

In this paper, we show that a finite sequence of vectors in 2-Hilbert space can be a 2-frames for the linear span of their elements, and introduce the optimal 2-frame bounds according to the frame operators.

Keywords: 2-inner product space, 2-frame, 2-frame bounds
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1 Introduction

Let H be a Hilbert space and I a set which is finite or countable. A collection $\{f_i\}_{i\in I}$ $\subseteq H$ is called a frame for H if there exist two constants $A, B > 0$ such that

$$A\|f\|^2 \leq \sum_{i\in I} |< f, f_i >|^2 \leq B\|f\|^2$$

for all $f \in H$. The constants A and B are called frame bounds. Frames have many applications in mathematics and engineering including wavelet theory, signal and image processing, operator theory, harmonic analysis and so on [5, 7]. A sequence satisfying the upper frame condition is called a Bessel sequence. For a frame $\{f_i\}_{i\in I}$ of H, the operator $T : \ell^2(\mathbb{N}) \to H$ defined by $Te_i = f_i$, $i \in \mathbf{N}$ is called the pre-frame operator. The frame operator $S = TT^*$ is defined by $S(f) = \sum_{i\in I} < f, f_i f_i$. A technique for representing the elements of a Hilbert space introduced by Duffin and Schaeffer [6] by frame theory. Nowadays frames work an alternative to orthonormal bases in Hilbert spaces which has many advantages [7]. In [1] A. A. Arefijamaal and Gh. Sadeghi have also introduced definition of 2-frame for a 2-inner product space and described some properties of them. First of all we recall the concept of 2-inner product space was first introduced by Y. J. Cho, et al, in [3].

Definition 1.1. Let X be a linear space of dimension greater than 1 over the field $\mathbf{F}$. Suppose that $(.,.|.)$ is a function from $X\times X\times X$ into $\mathbf{F}$ satisfying the following conditions:

(i) $(x, x|z) \geq 0$ and $(x, x|z) = 0$ iff x and z are linearly dependent; (ii) $(x, x|z) = (z, z|x)$;

(iii) $(y, x|z) = \overline{(x, y|z)}$;

(iv) $(\alpha x, x|z) = \alpha(x, x|z)$ for all every $\alpha \in \mathbf{F}$;

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