



Irreducible Smale spaces

Sarah Saeidi Gholikandi *

Tarbiat Modares university

Massoud Amini

Tarbiat Modares university

Abstract

Irreducible spaces play an important role in topological dynamical systems. There exists several equivalent definition for irreducible shift of finite type spaces which are the simplest Smale spaces. In this paper, we generalize them to Smale spaces and then get some results about the degree of factor maps on Smale spaces.

Keywords: Degree of factor maps, Irreducible spaces, Shift of finite type, Smale spaces

Mathematics Subject Classification [2010]: 37B10, 37D99

1 Introduction

1.1 Smale spaces

Definition 1.1. [1] A dynamical system is a pair (X, φ) where X is a topological space and φ is a homeomorphism of X .

Definition 1.2. [3] A dynamical system (X, φ) is said to be irreducible if, for every (ordered) pair of non-empty open sets U, V , there is a positive integer N such that $\varphi^N(U) \cap V$ is non-empty.

Definition 1.3. [3, 4] Suppose that (X, φ) is a compact metric space and φ is a homeomorphism of X . Then (X, φ) is called a Smale space if there exist constants ε_X and $0 < \lambda < 1$ and a continuous map from

$$\Delta_{\varepsilon_X} = \{(x, y) \in X \times X \mid d(x, y) \leq \varepsilon_X\}$$

to X (denoted with $[\cdot, \cdot]$) such that:

$$B\ 1 \quad [x, x] = x,$$

$$B\ 2 \quad [x, [y, z]] = [x, z],$$

$$B\ 3 \quad [[x, y], z] = [x, z],$$

$$B\ 4 \quad [\varphi(x), \varphi(y)] = [\varphi(x), \varphi(y)],$$

$$C\ 1 \quad d(\varphi(x), \varphi(y)) \leq \lambda d(x, y), \text{ whenever } [x, y] = y,$$

$C\ 2 \quad d(\varphi^{-1}(x), \varphi^{-1}(y)) \leq \lambda d(x, y), \text{ whenever } [x, y] = x, \text{ whenever both sides of an equation are defined.}$

*Speaker