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Lie structure of smash products *

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Abstract

We investigate the conditions under which the smash product of an (ordinary or restricted) enveloping algebra and a group algebra is Lie solvable or Lie nilpotent.

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1 Introduction

Let A be an associative algebra over a field and regard A as a Lie algebra via the Lie product defined by $[x, y] = xy - yx$, for every $x, y \in A$. Then, A is said to be Lie solvable (respectively, Lie nilpotent) if it is solvable (nilpotent) as a Lie algebra. The Lie structure of associative algebras have been extensively studied over the years and considerable attention has been especially devoted to group algebras (see e.g. [2, 3, 5]) and restricted enveloping algebras (see e.g. [6, 7, 8, 9, 10]).

Let G be a group and $\mathbb{F}$ a field. We denote by $\mathbb{F}G$ the group algebra of G over $\mathbb{F}$. We also denote by G' the derived subgroup of G. Passi, Passman and Sehgal established in [5] when $\mathbb{F}G$ is Lie solvable and Lie nilpotent.

Theorem 1.1 ([5])**.** Let $\mathbb{F}G$ be the group algebra of a group G over a field $\mathbb{F}$ of characteristic $p \geq 0$. Then $\mathbb{F}G$ is Lie nilpotent if and only if one of the following conditions hold:

1. $p = 0$ and G is abelian;
2. $p > 0$, G is nilpotent and G' is a finite p-group;

Theorem 1.2 ([5])**.** Let $\mathbb{F}G$ be the group algebra of a group G over a field $\mathbb{F}$ of characteristic $p \geq 0$. Then $\mathbb{F}G$ is Lie solvable if and only if one of the following conditions hold:

1. $p = 0$ and G is abelian;
2. $p > 2$ and G' is a finite p-group;
3. $p = 2$ and G has a subgroup N of index at most 2 such that N' is a finite 2-group.

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