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Talk A generalization of commutativity notion pp.: 1–4

A generalization of commutativity notion

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Abstract

Mason introduced the reflexive property for ideals. We in this article consider the reflexive ring property on nil ideals, introducing the concept of a *nil-reflexive* ring as a generalization of the reflexive ring property. It is proved that the polynomial and power series rings over right Noetherian (or NI) rings R are both shown to be nil-reflexive if $(aRb)^2 = 0$ implies $aRb = 0$ for all $a, b \in N(R)$. The structure of nil-reflexive rings is studied in relation to various sorts of ring extensions which have roles in ring theory.

Keywords: Nil-reflexive ring, Nil ideal, Polynomial ring, Power series ring, Right quotient ring
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1 Introduction

Throughout this article all rings are associative with identity unless otherwise specified. Given a ring R, the polynomial (resp., power series) ring with an indeterminate x over R is denoted by $R[x]$ (resp., $R[[x]]$). For any ring R and $n \geq 2$, denote the n by n full matrix ring over R by $Mat_n(R)$ and the n by n upper triangular matrix ring over R by $U_n(R)$. Let $D_n(R)$ denote the subring $\{A \in U_n(R) \mid \text{the diagonal entries of } A \text{ are all equal}\}$ of $U_n(R)$. We use $N^*(R)$ and $N(R)$ to denote the upper nilradical (i.e., the sum of all nil ideals) and the set of all nilpotent elements of R, respectively. It is well-known that $N^*(R) \subseteq N(R)$. $\mathbb{Z}$ ($\mathbb{Z}_n$) denotes the ring of integers (modulo n).

The reflexive property for right ideals was first studied by Mason [8]. a right ideal I of a ring R is called *reflexive* if $aRb \subseteq I$ implies $bRa \subseteq I$ for $a, b \in R$, and R is called *reflexive* if 0 is a reflexive ideal. Every semiprime ring is reflexive by an easy computation. Kwak and Lee [6] characterized the aspects of the reflexive and one-sided idempotent reflexive properties, and provided a method by which a reflexive ring, which is not semiprime, can always be constructed from any semiprime ring, and showed that the reflexive property is Morita invariant.

In [6], it is proved that a ring R is reflexive if and only if $IJ = 0$ implies $JI = 0$ for ideals I, J of R. We will consider the reflexive ring property on nil ideals of a ring.

Definition 1.1. A ring R is called *nil-reflexive* if $IJ = 0$ implies $JI = 0$ for nil ideals I, J of R.

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