# Existence results of three weak solutions for a two-point boundary value problem

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**Abstract**

we prove the existence of at least three weak solutions for one-dimensional fourth-order equations via two three critical points theorems.

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## 1 Introduction

In this note, we consider the following fourth-order boundary value problem

$$\begin{cases} u''''h(x,u') - u'' = [\lambda f(x,u) + g(u)]h(x,u'), & \text{in } (0,1), \\ u(0) = u(1) = 0 = u''(0) = u''(1), \end{cases} \tag{1}$$

where $\lambda$ is a positive parameter, $f : [0,1] \times \mathbb{R} \to \mathbb{R}$ is an $L^1$-Carathéodory function, $g : \mathbb{R} \to \mathbb{R}$ is a Lipschitz continuous function with the Lipschitz constant $L > 0$, i.e.,

$$|g(t_1) - g(t_2)| \leq L|t_1 - t_2|$$

for every $t_1, t_2 \in \mathbb{R}$, with $g(0) = 0$, and $h : [0,1] \times \mathbb{R} \to [0, +\infty)$ is a bounded and continuous function with $m := \inf_{(x,t)\in[0,1]\times\mathbb{R}} h(x,t) > 0$. Due to the importance of fourth-order two-point boundary value problems in describing a large class of elastic deflection, many researchers have studied the existence and multiplicity of solutions for such a problem, we refer the reader to [1, 4, 5] and references therein. In the present paper, employing two three critical points theorems, we establish the existence three weak solutions for the problem (1). We say that a function $u \in H^2([0,1]) \cap H_0^1([0,1])$ is a *weak solution* of problem (1) if

$$\int_0^1 u''(x)v''(x)\,dx + \int_0^1 \left( \int_0^{u'(x)} \frac{1}{h(x,\tau)} d\tau \right) v'(x)\,dx - \lambda \int_0^1 f(x,u(x))v(x)\,dx$$
$$- \int_0^1 g(u(x))v(x)\,dx = 0$$

holds for all $v \in H^2([0,1]) \cap H_0^1([0,1])$.

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