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(Amply) Rad-Supplemented Lattices

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Abstract

In this paper, we introduce and investigate (amply) Rad-supplemented lattices. If L is a Rad-supplemented lattice and $a \in L$, then $1/a$ is Rad-supplemented. It is shown that an algebraic lattice L is amply Rad-supplemented iff L is a Rad-supplemented. If $a/0$ and $1/a$ are Rad-supplemented and a has a Rad-supplement b in $d/0$ for every sublattice $d/0$ with $a \leq d$, then L is Rad-supplemented.

Keywords: Rad-Supplement, ample Rad-Supplement, Rad-Supplemented Lattice, amply Rad-Supplemented Lattice
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1 Introduction

Throughout this paper, we assume that L is a complete modular lattice with smallest element 0 and greatest element 1. An element a of a lattice L is called *small* in L (notation $a \ll L$), if $a \vee b \neq 1$ for every $b \neq 1$.

Let a and b be elements of a lattice L. a is called a *supplement* of b in L if a is minimal with respect to $1 = a \vee b$. a is a supplement of b in L iff $1 = a \vee b$ and $a \wedge b \ll a/0$ (see [3]). A lattice L is called *supplemented* if every element of L has a supplement in L. L is called *amply supplemented* if for any two elements a and b of L with $1 = a \vee b$, $b/0$ contains a supplement of a. A subset D of L is called *upper directed* if each finite subset of D has an upper bound in D. A lattice L is called *upper continuous* if $a \wedge (\bigvee D) = \bigvee_{d \in D}(a \wedge d))$ holds for every $a \in L$ and upper directed subset $D \subseteq L$. An element $a \in L$ is called *compact* if for every subset X of L and $a \leq \bigvee X$ there is a finite subset $F \subseteq X$ such that $a \leq \bigvee F$ and L is said to be compact if 1 is compact. A lattice L is called *algebraic* if each of its elements is a join of compact elements. An element $e \in L$ is called *essential* in L if $e \wedge a = 0$ holds for each element $a \in L$, $a \neq 0$. A lattice L is called *coatomic* if every proper element of L is contained in a maximal element of L. $Rad(L)$ will indicate *radical* of L (the intersection of all the maximal elements $\neq 1$ in L). We have the following properties of $Rad(L)$ in a lattice L.

Lemma 1.1. *[3, Lemma 7.8 and Proposition 12.2] Let a be an element in a lattice L.*
(1) $a \vee R(L) \leq R(1/a)$;
(2) If $a \leq R(L)$ then $R(1/a) = R(L)$;
(3) If L is algebraic, then $R(a/0) = a \wedge R(L)$.

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