



Amenability of weighted semigroup algebras based on a character

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Abstract

In this paper, we study ϕ -amenability and character amenability of weighted semigroup algebra $\ell^1(S, \omega)$. Indeed, we characterize character amenability of weighted semigroup algebras with a zero element. As an application, we give a characterization of character amenability of weighted Brandt semigroup algebras.

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1 Introduction

Let A be a Banach algebra and E is a Banach A -bimodule. We regard the dual space E' as a Banach A -bimodule with the following module actions:

$$(a \cdot f)(x) = f(x \cdot a) \quad , \quad (f \cdot a)(x) = f(a \cdot x) \quad (a \in A, f \in E', x \in E).$$

Kaniuth, Lau and Pym have introduced and studied in [6] and [7] the notion of ϕ -amenability for Banach algebras, where $\phi : A \rightarrow \mathbb{C}$ is a character. M. S. Monfared in [8] introduced and investigated the notion of character amenability for Banach algebras. Let $\Delta(A)$ be the set of all characters of the Banach algebra A , and let $\phi \in \Delta(A)$. A Banach algebra A is called left ϕ -amenable if for all Banach A -bimodules E for which the right module action is given by

$$x \cdot a = \phi(a)x \quad (x \in E, a \in A),$$

every continuous derivation $D : A \rightarrow E'$ is inner. We say that A is left character amenable if A is left ϕ -amenable for all $\phi \in \Delta(A)$ and has a bounded left approximate identity. Similarly, the right and two-sided version of ϕ -amenability and character amenability can be defined. These notions have been studied for various classes of Banach algebras. For more details see, [6], [7], [8].

Recently in [5], the authors studied the notions of ϕ -amenability and character amenability for the semigroup algebra $\ell^1(S)$, where S is a semilattice. Also, they characterized the character amenability of $\ell^1(S)$, where S is a uniformly locally finite inverse semigroup. As

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