A Modification of Adomian Decomposition Method to Delay Differential Equations Using Padé Approximation

Sepideh Ahmadzadeh* University of Shahrood
Alireza Nazemi University of Shahrood
Mehdi Ghovatmand University of Shahrood

Abstract

In this paper we present an application of technique combining Adomian Decomposition Method (ADM), Laplace transform and Padé approximant to find the analytical solutions for Delay Differential Equations (DDE). Solutions to DDEs are first obtained in convergent series form using the ADM. Then obtained from ADM's truncated series, we apply Laplace transform to it, then convert the transformed series into a meromorphic function by forming its Padé approximant. Finally, we take the inverse Laplace transform of the Padé approximant to obtain the analytical solution.

Keywords: Adomian decomposition method, Delay differential equation, Laplace transform, Padé approximant, Laplace-Padé resummation method
Mathematics Subject Classification [2010]: 44A10, 65Qxx, 74H15

1 Introduction

In this section we will explain the basic definitions of DDE, ADM, Padé approximant, Laplace-Padé resummation.

Definition 1.1: We define the nth order delay differential equations (DDE) of the form as follows:

$$u^{(n)}(x) = f(x, u(x), u(\eta_1(x)), u(\eta_2(x)), \cdots, u(\eta_m(x))), \quad x \in I = [0, a], \tag{1}$$

where $u : I \to R$, $f : I \times R^2 \to R$, $\eta_i : I(i = 1, 2, \cdots, m)$ and $\eta_i(x) < x$ for $x \in I$.

Definition 1.2: To introduce the basic idea of the ADM [**?**], we consider the operator equation $Fu = G$, where F represents a general nonlinear ordinary differential operator and G is a given function. Then F can be decomposed as:

$$Lu + Ru + Nu = G, \tag{2}$$

where N is a nonlinear operator, L is the highest-order derivative which is assumed to be invertible, R is a linear differential operator of order less than L and G is the nonhomogeneous term. The method is based by applying the operator L^{-1} formally to the expression (2) we obtain:

$$u = h + L^{-1}G - L^{-1}Ru - L^{-1}Nu, \tag{3}$$

where h is the solution of the homogeneous equation $Lu = 0$, with the initial-boundary conditions. The problem now is the decomposition of the nonlinear term Nu. To do this,

*Speaker