Combinatorically view to integral majorization

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Abstract

In this paper we look a little combinatorically to integral majorization. Integral majorization is a majorization relation on vectors with integer entries. We define for each path in a grid a vector, called vector grid, and then relate gird vectors to integral vectors and majorization. Then we propose some properties of these concepts and their relations.

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1 Introduction

Relations between combinatorics and linear algebra is very interesting and there are a lot of research in this area. Recently there are some researches on majorizations and combinatorics. For vectors $x, a \in \mathbb{R}^n$, we say that $x$is majoroized by a and denoted by $x \prec a$, provided that

$$\sum_{j=1}^{k} x_{[j]} \leq \sum_{j=1}^{k} a_{[j]}, \qquad \text{for} \quad k = 1, 2, \ldots, n-1,$$

and

$$\sum_{j=1}^{n} x_{[j]} = \sum_{j=1}^{n} a_{[j]},$$

where by $x_{[j]}$ we mean the j^{th} largest element of a vector x[3].

Let $M(a)$ be a polytope of all vectors majorized by a given vector $a \in \mathbb{R}^n$. A vector x is said integral vector if all of its elements are integer. For an integral vector a let $M_I(a)$ be the set of all integral vectors that are majorized by a. In [1] Dahl proposed an algorithm for computing combinatorically the cardinality of $M_I(a)$. Also Dahl propose a relation between $p(n)$, the number of different partitions of a natural number n, and majorization. $p(n)$ has been related to majorization [1]. Consider $(n, 0, 0, \ldots, 0)$ in $\mathbb{R}^n$. There are $p(n)$ nonincreasing nonnegative integral vectors in $\mathbb{R}^n$ that are majorized by $(n, 0, 0, \ldots, 0)$. In this paper we look more precisely to $M_I((n, 0, ..., 0))$ and $M_I((n, m, 0, ..., 0))$ and find some relations between $M_I(a)$ and grid paths.

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