Some results on the annihilator graph of a commutative ring

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Abstract

Let R be a commutative ring with identity, and let $Z(R)$ be the set of zero-divisors of R. The annihilator graph of R is defined as the undirected graph $AG(R)$ with the vertex set $Z(R)^* = Z(R) \setminus \{0\}$, and two distinct vertices x and y are adjacent if and only if $ann_R(xy) \neq ann_R(x) \cup ann_R(y)$. In this talk, some relations between annihilator graph and zero-divisor graph associated with a commutative ring are studied. Moreover, we give some conditions under which the annihilator graph and the zero-divisor graph associated with a ring are identical.

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1 Introduction

Recently, a major part of research in algebraic combinatorics has been devoted to the application of graph theory and combinatorics in abstract algebra. There are a lot of papers which apply combinatorial methods to obtain algebraic results in ring theory, see for example [1, 2, 3, 5] and [7].

Throughout this talk, all rings are assumed to be non-domain commutative rings with identity. We denote by $Nil(R)$ and $Z(R)$, the set of all nilpotent elements and the set of zero-divisors elements of R, respectively. Let $A \subseteq R$. The set of annihilators of A is denoted by $ann_R(A)$ and by A^*, we mean $A \setminus \{0\}$. The ring R is said to be *reduced*, if $Nil(R) = 0$. A prime ideal P of R is called an *associated prime ideal*, if $ann_R(x) = P$, for some non-zero element $x \in R$. The set of all associated prime ideals of R is denoted by $Ass(R)$.

Let $G = (V, E)$ be a graph, where $V = V(G)$ is the set of vertices and $E = E(G)$ is the set of edges. By $\overline{G}$, we mean the complement graph of G. The girth of a graph G is denoted by $gr(G)$. We write $u - v$, to denote an edge with ends u, v. A graph $H = (V_0, E_0)$ is called a *subgraph of* G if $V_0 \subseteq V$ and $E_0 \subseteq E$. Moreover, H is called an *induced subgraph by* V_0, denoted by $G[V_0]$, if $V_0 \subseteq V$ and $E_0 = \{\{u, v\} \in E \mid u, v \in V_0\}$. Let G_1 and G_2 be two disjoint graphs. The *join* of G_1 and G_2, denoted by $G_1 \vee G_2$, is a graph with the vertex set $V(G_1 \vee G_2) = V(G_1) \cup V(G_2)$ and edge set $E(G_1 \vee G_2) = E(G_1) \cup E(G_2) \cup \{uv \mid u \in V(G_1), v \in V(G_2)\}$. Also G is called a *null graph* if it has no edge. For a vertex x in G,

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