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On the Biclique Cover of Graphs

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Abstract

The *biclique cover number* $bc(G)$ of a graph G is the smallest number of bicliques of G such that every edge of G belongs to at least one of these bicliques. A *k-clique covering* of a graph G, is an edge covering of G by its cliques such that each vertex is contained in at most k cliques. The smallest k for which G admits a k-clique covering is called *local clique cover number* of G and is denoted by $lcc(G)$. In this paper, we find the relation between $bc(G)$ and $lcc(\overline{G})$ of the graphs. As a consequence, we show that if G is a graph with m edges such that $\overline{G}$ is a line graph then $bc(G) \leq 8 \ln m$.

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1 Introduction

Throughout the paper, all graphs are finite and simple graph. Let $V(G)$ denote the vertex set of the graph G and $E(G)$ denote its edge set. The complement $\overline{G}$ of the graph G is the simple graph whose vertex set is $V(G)$ and whose edges are the pairs of nonadjacent vertices of G. The term clique stands for the complete graph and biclique for the complete bipartite graph. The *biclique* (resp. *clique*) *cover number* $bc(G)$ (resp. $cc(G)$) of a graph G is the smallest number of bicliques (resp. cliques) of G such that every edge of G belongs to at least one of these bicliques (resp. cliques). A *k-biclique* (resp. *k-clique*)*covering* of a graph G, is an edge covering of G by its bicliques (resp. cliques) such that each vertex is contained in at most k bicliques (resp. cliques). The smallest k for which G admits a k-biclique (resp. clique) covering is called *local biclique* (resp. *clique*) *cover number* of G and is denoted by $lbc(G)$ (resp. $lcc(G)$). In the same manner, we can define biclique partition number $bp(G)$ and local biclique partition number $lbp(G)$, if we use partition instead of cover. These measures and its applications have been studied extensively throughout the literature; see [2, 3, 4, 5, 6]. Finding the relation between these parameters are also interesting and have been studied in the literature; see [8]. In [8], it has been shown that $bp(G)$ can be bounded in term of $bc(G)$, in particular, they have shown that $bp(G) \leq \frac{1}{2}(3^{bc(G)} - 1)$. However, they showed that the analogous result does not hold for the local measures. In this paper, we find a relation between $bc(G)$ and $lcc(\overline{G})$. In particular, we show that if G is a graph with m edges then $bc(G) \leq \frac{1}{2}4^{lcc(\overline{G})} \ln m$. Finding

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