A generalization of paracontact psudo-metric manifolds

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Abstract

In this paper, we give a characterization of a paracontact psudo-metric and K-paracontact psudo-metric manifolds as a special almost paracontact psudo-metric manifold and introduce a quasi paracontact psudo-metric manifold according to quasi para Kähler psudo-metric manifold which is a natural generalization of the paracontact psudo-metric manifolds.

Keywords: Almost paracontact, Paracontact, Psudo-metric, Quasi para Kähler
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1 Introduction

A systematic study of general paracontact pseudo-metric structures was undertaken by Simeon Zamkovoy in 2009 [8]. Cruceanu, Fortuny and Gadea gave the survey article on paracomplex geometry in 1996 [2] and Mykhaylo Chursin, Lars Schafer and Knut Smoczyk defined quasi para Kähler manifolds in 2010 [3]. In this paper, we introduce a quasi paracontact psudo-metric manifold according to quasi para Kähler psudo-metric manifold which is a natural generalization of the paracontact psudo-metric manifolds. Now, let (M, ϕ, ξ, η, g) be a $(2n+1)$-dimensional almost paracontact pseudo-metric manifold and $\overline{M} = M \times \mathbb{R}$ be the product manifold of M and a real line $\mathbb{R}$ equipped with the following almost para Hermitian structure $(\overline{J}, \overline{g})$ defined by

$$\begin{aligned} &\overline{J}X = \phi X + \eta(X)\frac{\partial}{\partial t}, \quad \overline{J}\frac{\partial}{\partial t} = \xi, \\ &\overline{g}(X,Y) = e^{-2t}g(X,Y), \quad \overline{g}(X, \frac{\partial}{\partial t}) = 0, \quad \overline{g}(\frac{\partial}{\partial t}, \frac{\partial}{\partial t}) = -e^{-2t}, \end{aligned} \tag{1}$$

for $X, Y \in \chi(M)$ and $t \in \mathbb{R}$. Now, we denote by $\overline{\nabla}$ the covariant derivative with respect to the metric $\overline{g}$ on $\overline{M}$. Then, from (1) by direct calculation, we have

$$\begin{aligned} &\overline{\nabla}_X Y = \nabla_X Y + g(X,Y)\frac{\partial}{\partial t}, \quad \overline{\nabla}_{\frac{\partial}{\partial t}} X = -X, \\ &\overline{\nabla}_X \frac{\partial}{\partial t} = -X, \quad \overline{\nabla}_{\frac{\partial}{\partial t}} \frac{\partial}{\partial t} = -\frac{\partial}{\partial t}, \end{aligned} \tag{2}$$

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