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Semigroups with apartness: constructive versions of some classical theorems

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Abstract

The starting point of our work is the structure $(S, =, \neq, \cdot\,)$ called a semigroup with apartness. We examine and prove constructive analogues of some classical theorems, like, for example, isomorphism theorems and Cayley's theorem.

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1 Introduction

Following [10, Vol II], "The study of algebraic structures in an intuitionistic setting was undertaken by Heyting [7]." Within **BISH**, which forms the framework for our work, the history of constructive semigroups with an inequality began recently, [1]. In [3], [9] it is shown/announced that constructive algebraic structures with apartness can be applied in computer science (especially in computer programming) as well.

Definition 1.1. By an **apartness** on S (see [8]), we mean a binary relation $\not\backsimeq$ on S which satisfies the axioms of irreflexivity, symmetry and cotransitivity: $\neg(x \not\backsimeq x)$, $x \not\backsimeq y \;\Rightarrow\; y \not\backsimeq x$, $x \not\backsimeq z \;\Rightarrow\; \forall_y\,(x \not\backsimeq y \vee y \not\backsimeq z)$. We then say that $(S, \backsimeq, \not\backsimeq)$ is a **set with apartness**. An apartness is **tight** if $\neg(x \not\backsimeq y) \Rightarrow x \backsimeq y$.

Definition 1.2. Let $(A, \backsimeq, \not\backsimeq\,)$ be a set with apartness. A function $f : A \longrightarrow A$ is *strongly extensional*, or, for short, a *se-function* if whenever we have $f(a) \not\backsimeq f(b)$, then $a \not\backsimeq b$ follows, $a, b \in A$.

Following [6], [10], where the notion of commutative constructive semigroups with tight apartness has appeared, we define and put the notion of noncommutative constructive semigroups with "ordinary" apartness in the centre of our study.

Definition 1.3. A tuple $(S, \backsimeq, \not\backsimeq, \cdot)$ is a **semigroup with apartness** with $(S, \backsimeq, \not\backsimeq)$ as a set with apartness, $\cdot$ a binary operation on S which is associative, i.e. $\forall_{a,b,c\in S}\ [(a \cdot b) \cdot c \backsimeq a \cdot (b \cdot c)]$, and strongly extensional, i.e. $\forall_{a,b,x,y\in S}\ (a \cdot x \not\backsimeq b \cdot y \Rightarrow (a \not\backsimeq b \vee x \not\backsimeq y))$.

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