



## On the planar, outer planar and end-regular zero divisor graph of the ring $C(X)$

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### Abstract

In this paper, we study the planarity, outerplanarity, and end-regularity of the zero-divisor graph of the ring of all real valued continuous functions  $C(X)$ , which is denoted by  $\Gamma(C(X))$ . Also, by using the ring properties of  $C(X)$ , the graph properties of  $\Gamma(C(X))$ , and the topological properties of  $X$ , we investigate the end-regularity of the graph  $\Gamma(C(X))$ .

**Keywords:** Zero divisor graph, The ring of continuous functions, Planar graph, Outerplanar graph, End-regular graph

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## 1 Introduction

The idea of a zero-divisor graph of a commutative ring was first introduced by I. Beck [2] in 1988, where he was mainly interested in coloring. This investigation of colorings of a commutative ring was then continued by D. D. Anderson and M. Naseer. Their definition was slightly different than ours; they let all elements of ring be vertices and distinct vertices  $x$  and  $y$  are adjacent if and only if  $xy = 0$ . Anderson and Livingston introduced and studied the zero-divisor graph whose vertices are the non-zero zero-divisors.

Let  $C(X)$  be the ring of all real valued continuous functions on a completely regular Hausdorff space  $X$ . By the zero divisor graph  $\Gamma(C(X))$  of  $C(X)$  we mean the graph with vertices consists of all nonzero zero-divisors of  $C(X)$  such that there is an edge between distinct vertices  $f$  and  $g$  if and only if  $fg = 0$ .

In this paper, we determine the planarity, outerplanarity, and end-regularity of  $\Gamma(C(X))$  by using the ring properties of  $C(X)$ , the graph properties of  $\Gamma(C(X))$ , and the topological properties of  $X$ . Also, we show that, in some cases, the graph  $\Gamma(C(X))$  is not end-regular.

## 2 Main results

In this section, we first state some preliminaries from the ring  $C(X)$ , topology and graph theory which are expected to be useful in this paper. We use the standard terminology from [3] and [4].

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