Septic B-spline solution of one dimensional Cahn-Hillird equation

Reza Mohammadi*
University of Neyshabur

Fatemeh Borji
University of Neyshabur

Abstract

The septic B-spline collocation scheme is implemented to find numerical solution of one dimensional Cahn-Hillird equation. The scheme is based on the finite-difference formulation for time integration and septic B-spline functions for space integration. Stability and Convergence of the scheme are discussed. The accuracy of the proposed method is demonstrated a test problem.

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1 Introduction

Consider the one-dimensional Cahn-Hilliard equation

$$\frac{\partial u}{\partial t} + \gamma \frac{\partial^4 u}{\partial x^4} - \frac{\partial^2 \varphi(u)}{\partial x^2} = 0, \quad x \in (a, b), \quad t \geq 0, \tag{1}$$

with initial condition

$$u(x, 0) = \phi(x), \quad x \in [a, b], \tag{2}$$

and the boundary conditions

$$\frac{\partial u(a, t)}{\partial x} = \frac{\partial u(b, t)}{\partial x} = 0, \quad \frac{\partial^3 u(a, t)}{\partial x^3} = \frac{\partial^3 u(b, t)}{\partial x^3} = 0, \quad t \geq 0, \tag{3}$$

where $\varphi(u) = \frac{d\psi(u)}{du}$ and $\psi(u) = \frac{1}{4} r_2 u^4 + \frac{1}{3} r_1 u^3 + \frac{1}{2} r_0 u^2$. The constant γ is positive, and r_0, r_1, r_2 are given constants. It is known if the initial data $u_0 \in H_E^2([a, b]) = \{f \in H^2([a, b]) : \frac{\partial f}{\partial x} = 0 \text{ on } a \text{ and } b\}$ then the problem (1)-(3) has a unique solution for all times [1].There are many algorithms for numerical solution of the C-H equations in literature, using different methods (for example see references [2, 3, 4]).

In current work, we will use septic B-spline to solve the Cahn-Hillird partial differential equation (1). The main purpose is to analyze the efficiency of the septic B-spline-difference method for such problems with sufficient accuracy. The time derivative is replaced by horizontal method of line finite-difference representation and the space derivatives by septic B-spline. In comparison with the existing well-known methods, our method is simple with better numerical stability and lower computational cost. Numerical computations show that our results are well accepted.

*Speaker