Some properties on non-abelian tensor product of groups

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Abstract

we prove that the non-abelian tensor product $G \otimes H$ is locally nilpotent or locally solvable if such information is given in terms of $D_H(G)$, the derivative subgroup of G afforded by the action of H on G. This derivative subgroup reduce to the commutator subgroup G' of G if $G = H$ and the actions are conjugation. Also we present a survey of results into the tensor analogues of 3-Engel groups. Finally, we present some results about subgroup which is generalization of the tensor analogue of right 2-Engel elements of a group.

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1 Introduction

For any group G, the *the nonabelian tensor square* is a group generated by the symbols $g \otimes h$, subject to the relations,

$$gg' \otimes h = (g^{g'} \otimes h^{g'})(g' \otimes h) \ \ and \ \ g \otimes hh' = (g \otimes h')(g^{h'} \otimes h^{h'})$$

where $g, g', h, h' \in G$ and $g^h = h^{-1}gh$.
The nonabelian tensor square is a special case of the nonabelian tensor product which has its origins in homotopy theory. It was introduced by Brown and Loday in [3] and [4], extending ideas of Whitehead in [10]. In [2], Brown, Johnson, and Robertson start the investigation of nonabelian tensor squares as group theoretical objects. If $G = H$ and all actions are given by conjugation, then $G \otimes G$ is called the non-abelian tensor square. One notes that the non-abelian tensor square of a given group always exists.

Definition 1.1. Let G and H be groups with H acting on G. Then the subgroup

$$D_H(G) = \langle g^{-1}g^h \mid g \in G, h \in H \rangle$$

of G is the *derivative of G by H*. The derivative H by G, $D_G(H)$, is defined as similar. Taking $G = H$, and all actions to be conjugation, the derivative subgroup reduce to the commutator subgroup G' of G.

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