Monotonicity and dominated best proximity pair in Banach lattices and some applications

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Abstract

In this paper we introduce the dominated Best proximity pair problem in Banach lattices. We give some necessary and sufficiency conditions such that this problem is uniquely solvable in STM space. Also we show that every UM spaces have property UC in Banach Lattices.

Keywords: Banach Lattice, Best proximity pair, STM Space, Property UC.
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1 Introduction

Let $(X, \leq)$ be a Banach lattice and A, B be two nonempty subset of $X$and T be a mapping from A in to B. $x \in A$ is called a point of best proximity pair if $\|x - Tx\| = d(A, B)$ where

$$d(A, B) = \inf\{\|x - y\| : (x, y) \in A \times B\}.$$

The set of all best proximity points is denoted by T_A^B. T is called a nonexpansive map if $\|Tx - Ty\| \leq \|x - y\|$ for each $x, y \in A$. Best proximity pair also evolves a generalization of the concept of fixed point of mapping. Indeed every best proximity pair is a fixed point of T, whenever $A \cap B \neq \emptyset$. The problem of best proximity pair is discussed by many authors for more information you can refer to [2], [3], [9] and [10]. Eldered and Veeramani in [3] proved that for a cyclic contraction map in a uniformly convex Banach space there exists a unique best proximity pair and Sankar Raj and Veeramani proved similarly results for relatively nonexpansive map. In [10] Suzuki et.al by using Lemma 3.8 in [3] defined property UC and discussed the existence of best proximity pair. In this paper we introduce the concept of dominated best proximity pair and stated some condition to guaranteed the existence of best proximity pair. For general information in Banach lattices we can refer to chapter one of [1] and [7].

Definition 1.1. [6] A Banach lattice X is said to be strictly monotone ($X \in$ STM) if for all $x, y \in X^+$, the conditions $x \geq y,\ y \neq 0$ and $\|x\| = \|y\|$ implies $x = y$.

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