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Hyers-Ulam-Rassias stability of functional equations on quasi-normed liner spaces

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Abstract

In this paper, we use the definition of quasi-normed spaces and the behaviors of solutions of the additive functional equation are described. The Hyers-Ulam stability problem of this equation is discussed and theorems concerning the Hyers-Ulam-Rassias stability of the equation are proved on quasi-normed linear space.

Keywords: Complte quasi-normed liner space, Functional equation, Quasi-morm.
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1 Introduction

Defining, in some way, the class of approximate solutions of the given functional equation one can ask whether each mapping from this class can be somehow approximated by an exact solution of the considered equation. Such a problem was formulated by Ulam in 1940 (cf. [4]) and solved in the next year for the Cauchy functional equation by Hyers [2]. In 1950, Aoki [1] and in 1978, Rassias [3] proved a generalization of Hyers theorem for additive and linear mappings, respectively:
The result of Rassias has influenced the development of what is now called the Hyers-Ulam-Rassias stability theory for functional equations.

2 Main results

Definition 2.1. Let X is a vector space over F and $k \geq 1$. Furthermore, let $\|.\|_k : X \longrightarrow [0, \infty)$ be a function such that for all $x, y \in X$ and $c \in F$:
(i) $\|cx\|_k = |c|\|x\|_k$,
(ii) $\|x + y\|_k \leq k(\|x\|_k + \|y\|_k)$,
(iii) $\|x\|_k = 0$ if and only if $x = 0$.

Theorem 2.2. *Let $f : X \longrightarrow Y$ be a function between complete quasi-normed linear spaces such that*

$$\|f(x + y) - f(x) - f(y)\|_k \leq \delta, \text{ for all } x, y \in X,$$

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