Space of Operators

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Abstract

The purpose of this work is to give some new results concerning space of operators in terms of some subsets of Banach space X. We will give equivalent characterization of Banach spaces X in which every V^*-subset of X is relatively compact. We also discuss some applications of these results to the subspaces of bounded linear operators.

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1 Introduction

Throughout this talk, X and Y will denote real Banach spaces. A bounded subset A of X is called a *Dunford-Pettis* (DP) (resp. *limited*) subset of X if

$$\lim_n(\sup\{\mid x_n^*(x)\mid : x \in A\}) = 0$$

for each weakly null (resp. w^*-null) sequence (x_n^*) in X^*.

A bounded subset S of X is said to be *weakly precompact* provided that every sequence from S has a weakly Cauchy subsequence. The unit ball of a Banach space X is weakly precompact if and only if X does not contain copies of ℓ_1 (by Rosentlal's ℓ_1 theorem). Every Dunford-Pettis set is weakly precompact , e.g., see [12], p. 377, [1], [8]. We note that every relatively compact subset of X is limited and every limited subset of X is Dunford-Pettis. Thus every relatively compact subset of X is DP.

A Banach space X has the *Gelfand-Phillips* (GP) *property* if every limited subset of X is relatively compact. The Banach space X has the *Dunford-Pettis relatively compact property* ($DPrcP$) (resp. the RDP^* *property*) if every Dunford-Pettis subset of X is relatively compact (resp. relatively weakly compact) [3], [7] . Certainly, if a Banach space X has the $DPrcP$, then X has the (GP) property (since any limited set is a DP set). Note that every Schur space has the $DPrcP$.

Closely related to the notions of DP sets and limited sets is the idea of an L-set, e.g., see Bator [2] and Emmanuele [5], [6]. A Bounded subset A of X^* is called an *L-subset* of X^* if

$$\lim_n(\sup\{\mid x^*(x_n)\mid : x^* \in A\}) = 0$$

for each weakly null sequence (x_n) in X. Emmanuele and Bator [5], [2] showed that $\ell_1 \not\hookrightarrow X$ iff any L-subset of X^* is relatively compact iff X^* has the $DPrcP$.