Autoregressive Hilbertian Processes

Maryam Hashemi
Department of statistics, University of Isfahan, Khansar Unit, Isfahan, Iran

Abstract

In this article, We consider autoregressive processes in Hilbert spaces. We present here existence, the strong law of large numbers and estimation of autocovariance operators.

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1 Introduction

functional data often arise from measurements obtained by separating an almost continuous time record into natural consecutive intervals, for examples days. many important examples of data that can be naturally treated as functional come from financial records. The price of an asset exists only when the asset is traded. A great deal of financial research has been done using the closing daily price, i.e. the price in the last transaction of a trading day. However many assets are traded so frequently that one can practically think of a price curve that is defined at any moment of time.[2]
The Hilbertian autoregressive model of order 1 (ARH(1)) generalizes the classical AR(1) model to random elements with values in Hilbert spaces. This model was introduced by Bosq (2000), then studied by several authors, as Mourid (1993), Besse and Cardot (1996), Pumo (1999), Mas (2002, 2007), Horvath et al. (2010). Bosq in his fundamental work (2000) provides basic results on Hilbertian strongly second order autoregressive and moving average processes. The existence, covariance structure, parameter estimation, strong law of large numbers and central limit theorem, are the topics that are covered by Bosq (2000).

For writing definition, theorem, proof, throughout this paper, we consider H as a real separable Hilbert space equipped with scalar product $\langle .,. \rangle$, norm $\| \cdot \|$ and Borel σ-field $\mathcal{B}$. The H-valued random variables considered below are defined over the same probability space $(\Omega, \mathcal{F}, P)$ supposed to be rich enough and complete.

Definition 1.1. A sequence $X = \{X_n, n \in \mathcal{Z}\}$ of H-random variables is called an autoregressive Hilbertian process of order 1 (ARH(1)) associated with (μ, ϵ, ρ) if it is stationary and such that

$$X_n - \mu = \rho(X_{n-1} - \mu) + \epsilon_n, \quad n \in \mathcal{Z} \tag{1}$$

where $\epsilon = \{\epsilon_n, n \in \mathcal{Z}\}$ is an H-white noise, $\mu \in H$, and $\rho \in \mathcal{L}$.