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Divisibility Graph for some finite simple groups*

Adeleh Abdolghafourian† Yazd University
Mohammad A. Iranmanesh Yazd University
Alice C. Niemeyer Maynooth University

Abstract

The *divisibility graph* of a finite group G has vertex set the conjugacy class sizes of non-central elements in G and two vertices are adjacent if one divides the other. We determine the connected components of the divisibility graph of the finite simple groups of Lie type over a finite field of odd characteristic.

Keywords: Conjugacy class, Divisibility graph, Finite simple group, Prime graph.
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1 Introduction

In [3] the *divisibility graph* which is related to a set of positive integers have been introduced. The divisibility graph, $\overrightarrow{\mathrm{D}}(X)$ is a graph with vertex set $X^* = X \setminus \{1\}$ and there is an arc between two vertices a and b if and only if a divides b. It is also asked for the structure and especially the number of connected components of this graph (see [3, Question 7]).

Let G be a finite group and $\mathrm{cs}(G)$ denotes the set of conjugacy class sizes of non-central elements in G. We show the underlying graph of $\overrightarrow{\mathrm{D}}(\mathrm{cs}(G))$ by $\mathrm{D}(G)$ without changing the name for convenience. Actually by the *divisibility graph* $\mathrm{D}(G)$ we mean a graph with vertex set $\mathrm{cs}(G)$ and two conjugacy class sizes are adjacent if one divides the other.

In [1], The structure of divisibility graph $\mathrm{D}(G)$, where G is a symmetric group or an alternating group is studied.

Theorem 1.1. *[1, Corollary 11] $D(S_n)$ has at most two connected components. If it is disconnected then one of its connected components is K_1.*

Theorem 1.2. *[1, Corollary 17] $D(A_n)$ has at most three connected components. If it is disconnected, then two of its connected components are K_1.*

Also in [2], the structure of divisibility graphs for $\mathrm{PSL}(2,q)$, $\mathrm{Sz}(q)$ and 26 sporadic simple groups have been described.

Theorem 1.3. *[2, Theorem 2.1] Let $G = \mathrm{PSL}(2,q)$. Then $\mathrm{D}(G)$ is either $3\mathrm{K}_1$ or $\mathrm{K}_2{+}2\mathrm{K}_1$.*

Theorem 1.4. *[2, Theorem 2.2] Let $G = \mathrm{Sz}(q)$. Then $\mathrm{D}(G) = \mathrm{K}_2 + 3\mathrm{K}_1$.*

*Will be presented in English
†Speaker