Congruence on a ternary monoid generated by a relation

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Abstract

In this paper we define the notion of congruence on a ternary monoid generated by a relation and we determine the method of obtaining a congruence on a ternary monoid T from a relation R on T. Making of congruences is important because we can gain new ternary monoid from them.

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1 Introduction

The theory of ternary algebraic systems was introduced by D. H. Lehmer [3] in 1932, but before that (1904) such structures were studied by E. Kanser [2] who gave the idea of n-ary algebras. Lehmer studied certain ternary algebraic systems called triplexes, commutative ternary groups, in fact. Ternary structures and their generalization, the so called n-ary structures, are outstanding for their application in physics. The notion of congruence was first introduced by Karl Fredrich Gauss in the beginning of the nineteenth century. Congruences are a special type of equivalence relations which play a vital role in the study of quotiont structures of different algebraic structures. In this paper we define the notion of congruence on a ternary monoid generated by a relation and we determine the method of obtaining a congruence on a ternary monoid T from a relation R on T. Making of congruences is important because we can gain new ternary monoid (in fact quotiont monoids) from them. The first we express some primary notions.

Definition 1.1. A non-empty set T is called a ternary semigroup if there exists a ternary operation $T \times T \times T \to T$, written as $(a, b, c) \to abc$ satisfying the following statement: $(abc)de = a(bcd)e = ab(cde)$ for all $a, b, c, d, e \in T$.

Definition 1.2. An element e of a ternary semigroup T is called,
(i) a left identity (left unital element) if $eex = x$ for all $x \in T$;
(ii) a right identity (right unital element) if $xee = x$ for all $x \in T$;
(iii) a lateral identity (lateral unital element) if $exe = x$ for all $x \in T$;
(iv) a two-sided identity (bi-unital element) if $eex = xee = x$ for all $x \in T$;
(v) an identity (unital element) if $eex = exe = xee = x$ for all $x \in T$.

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