Edge group choosability of planar graphs with maximum degree at least 11

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Abstract

A graph G is edge-k-group choosable if its line graph is k-group choosable. In this paper, we present an edge-group choosability version of Vizing's conjecture and we shall show that it is true for graphs with maximum degree less than 4 and for planar graphs with maximum degree at least 11.

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1 Introduction

We consider only simple graphs. For a graph G, we denote its vertex set, edge set, minimum degree, maximum degree, and line graph by $V(G)$, $E(G)$, $\delta(G)$, $\Delta(G)$, and $\ell(G)$, respectively. Let $d_G(x)$, or simply $d(x)$, denote the degree of a vertex x in G. A *plane graph* is a particular drawing of a planar graph in the Euclidean plane. A k*-coloring* of a graph G is a mapping ϕ from $V(G)$ to the set of colors $\{1, 2, \ldots, k\}$ such that $\phi(x) \neq \phi(y)$ for every edge xy. A graph G is k*-colorable* if it has a k-coloring. The *chromatic number* $\chi(G)$ is the smallest integer k such that G is k-colorable. A mapping L is said to be a *list assignment* for G if it supplies a list $L(v)$ of possible colors to each vertex v. A k*-list assignment* of G is a list assignment L with $|L(v)| = k$ for each vertex $v \in V(G)$. If G has some k-coloring ϕ such that $\phi(v) \in L(v)$ for each vertex v, then G is L*-colorable* or ϕ is an L*-coloring* of G. We say that G is k*-choosable* if it is L-colorable for every k-list assignment L. The *choice number* or *list chromatic number* $\chi_l(G)$ is the smallest k such that G is k-choosable. By considering colorings for $E(G)$, we can define analogous notions such as *edge-*k*-colorability*, *edge-*k*-choosability*, the *chromatic index* $\chi'(G)$, the *choice index* $\chi'_l(G)$, etc. Clearly, we have $\chi'(G) = \chi(\ell(G))$ and $\chi'_l(G) = \chi_l(\ell(G))$. The notion of list coloring of graphs has been introduced by Erdős, Rubin, and Taylor [5] and Vizing [13]. The following conjecture, which first appeared in [1], is well-known as the List Edge Coloring Conjecture.

Conjecture 1. If G is a multi-graph, then $\chi'_l(G) = \chi'(G)$.

Although Conjecture 1 has been proved for a few special cases such as bipartite multigraphs [6], complete graphs of odd order [7], multicircuits [15], graphs with $\Delta(G) \geq 12$ that

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