Solving Large Sparse Linear Systems by Using QR-Decomposition whit Iterative Refinement

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Abstract

In this article, for solution of a system of linear algebraic equations $Ax = b$ with a large sparse coefficient matrix A, the QR-decomposition with iterative refinement (QRIR) is compared with the QR-decomposition by means of Givens rotations(QRGR), which is without iterative refinement and leads to direct solution. We verify by numerical experiments that the use of sparse matrix techniques with QRIR may result in a reduction of both the computing time and the storage requirements.

Keywords:large sparse linear systems, QR-decomposition with Givens rotations(QRGR), QR-decomposition with iterative refinement(QRIR)

1 Introduction

A system of linear algebraic equations is

$$Ax = b \tag{1}$$

where A is a nonsingular, large, sparse and nonsymmetric matrix of order n and b is a given column vector of order n. To solve the linear system (1) one can try several different algorithms. One method is to find the inverse and multiply it on both sides, which is expensive computationally. Another method is to make a guess of the solution and iteratively refine that guess until the error is suitably small. The method proposed here is an iterative refinement based on the QR-decomposition method. The QR-decomposition of a matrix is a decomposition of a matrix A into a product $A = QR$ of an orthogonal matrix Q and an upper triangular matrix R. There are several methods for actually computing the QR-decomposition, such as the Gram-Schmidt process, Householder transformations, or Givens rotations. Householder transformation has greater numerical stability than the Gram-Schmidt method. Givens rotation procedure is used here, which does the equivalent of the sparse Givens matrix multiplication, without the extra work of handling the sparse elements. The Givens rotation procedure is useful in situations where only a relatively few off diagonal elements need to be zeroed, and is more easily parallelized than Householder transformations. The factorization operation count with Givens rotation is always smaller than other methods. In this paper for computing the QR-decomposition, we use Givens rotations algoritm for sparse matrices[2].

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