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Some equivalent conditions to strong uniqueness in normed linear space

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Abstract

In this work we investigate equivalent condition for strong unique best approximation and its uniqnuess and also strongly unique. Also, for finite dimensional subspace of $C(X, \mathbb{R})$, Lipschitz continuity of order 1 and strong uniqueness of order 1 are essentially equivalent.

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1 Introduction

Let X be a finite set with the discrete topology and $C(X, \mathbb{R}^k)$ be the space of vector-valued functions from X to k-dimensional Euclidean space $\mathbb{R}^k$. A norm for functions in $C(X, \mathbb{R}^k)$ is defined as follows:

$$\|f\| := \max_{x\in X} \|f(x)\|_2,$$

where $\|.\|_2$ denotes the Euclidean norm on $\mathbb{R}^k$.

Definition 1.1. Let G be a nonempty subset of a normed linear space X and let $x \in X$. An element $y_0 \in G$ is called a best approximation, or nearest point to x from G, if

$$\|x - y_0\| = \mathrm{d}(x, G),$$

where $\mathrm{d}(x, G) = \inf_{y\in G} \|x - y\|$. The number $\mathrm{d}(x, G)$ is called the distance from x to G, or the error in approximating x by G.

The set (possibly empty) of all best approximation from x to G is denoted by $P_G(x)$, i.e.

$$P_G(x) := \{y \in G | \ \mathrm{d}(x, G) = \|x - y\|\}.$$

This defines a mapping P_G from X into the subsets of G called the metric projection onto G.

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