

46th Annual Iranian Mathematics Conference

25-28 August 2015

Yazd University



Some properties of the character graph of a solvable group

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Abstract

Let G be a finite solvable group. In this paper we consider the character graph of G and study some parameters of this graph. At first, we answer this question that when is this graph Hamiltonian? Then we obtain conditions which it is a complete graph. Finally, we study the coloring of this graph.

Keywords: Character graph, Solvable group, Hamiltonian graph, Complete graph. Mathematics Subject Classification [2010]: 20E45, 20C15

1 Introduction

Let G be a finite group, and let cd(G) be the set of all character degrees of G, that is, $cd(G) = \{\chi(1) | \chi \in Irr(G)\}$, where Irr(G) is the set of all complex irreducible characters of G. The set of prime divisors of character degrees of G is denoted by $\rho(G)$. It is well known that the character degree set cd(G) may be used to provide information on the structure of the group G. For example, Ito-Michler's Theorem [8] states that if a prime p divides no character degree of a finite group G, then G has a normal abelian Sylow p-subgroup. Another result due to J. Thompson [10] says that if a prime p divides every non-linear character degree of a group G, then G has a normal p-complement.

A useful way to study the character degree set of a finite group G is to associate a graph to cd(G). One of these graphs is the character graph $\Delta(G)$ of G. Its vertex set is $\rho(G)$ and two vertices p and q are joined by an edge if the product pq divides some character degree of G. We refer the readers to a survey by Lewis [5] for results concerning this graph and related topics. When G is a solvable group, some interesting results on the character graph of G have been obtained. For example, Manz in [6] has proved that in this case, $\Delta(G)$ has at most two connected components. Manz, Willems and Wolf in [7] have proved that diameter of $\Delta(G)$ is at most 3. If $\Delta(G)$ is regular with n vertices. Morresi Zuccari in [9] proved that $\Delta(G)$ is either complete or (n-2)-regular graph. Moreover, if $\Delta(G)$ is (n-2)-regular and G has no normal non-abelian Sylow subgroups, he shown that G is a direct product of groups having disconnected character graph.

Throughout this work all groups are assumed to be finite and all graphs are simple and finite. Here we bring some definitions and notations from [1].

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