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Perfect dimension

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Abstract

In this article, we introduce and study the concept of perfect dimension, which is a Krull like dimension extension of the concept of DCC on finitely generated submodules or being perfect. We show that some of the basic results of Krull dimension is true for perfect dimension.

Keywords: finitely generated module, Krull dimension, Perfect dimension, Distributive module.

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1 introduction

Lemonnier [6] has introduced the concept of deviation of an arbitrary poset, in particular, when applied to the lattice of all submodules of a module ${}_RM$, give the concept of Krull dimension (in the sense of Rentschler and Gabriel) see [5, 3, 8]. The Krull dimension of an R-module is denoted by $k\text{-}\dim M$. It is well known that an R-module M is perfect if and only if it satisfies the descending chain condition (DCC) on finitely generated submodules. Motivated by this fact, one is tempted to extend this for Krull dimension. Let us give a brief outline of this paper. Section 1, is the introduction. In section 2, of this paper we study the concept of perfect dimension of an R-module M, denoted by $p\text{-}\dim M$, which is the deviation of $F(M)$, the poset of finitely generated submodules of M. It is also denoted by $K(F(M))$ in [1]. We investigate some basic properties of perfect dimension. It is manifest that if $k\text{-}\dim M$ exists, then $p\text{-}\dim M \leq k\text{-}\dim M$, where M is an R-module. We observe that for any ordinal number α, there exists an R-module M such that $p\text{-}\dim M = \alpha$ but it does not have Krull dimension. It is proved that if M is a perfect R-module and for each small submodule N of M, $\frac{M}{N}$ has finite Goldie dimension, then M is Artinian. Consequently we prove that over perfect rings R, any quotient finite dimensional module M is Artinian. We give another proof for [1, Proposition 1.17]. Consequently we observe that if an R-module M has perfect dimension and for each essential submodule E of M, $\frac{M}{E}$ has finite Goldie dimension, then either M has a non-finitely generated socle or $p\text{-}\dim M = k\text{-}\dim M$. We recall that an R-module M

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