some starlike and convex properties for Hypergeometric functions *

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Abstract

In this paper, we finding conditions on the triplet (a, b, c) so that the function $zF(a, b; c; z)$ is starlike in Δ, where $F(a, b; c; z)$ denotes the hypergeometric function. Also the geometric problem of starlikeness and close to convexity of $zF(a, b; c; z)$ is studied.

Keywords: Starlike, Convex , univalent, Hypergeometric functions.
Mathematics Subject Classification [2010]: 30C45, 30C55

1 Introduction

let S denote the class of all functions f of the form

$$f(z) = z + \sum_{n=0}^{\infty} a_n z^n \tag{1}$$

that are analytic and univalent in the open unit disk $\Delta = \{z \in C : \mid z \mid < 1\}$.

Definition 1.1. A function $f \in S$ is said to be starlike of order $\beta(0 \leq \beta < 1)$ if and only if $Re\left(\frac{zf'(z)}{f(z)}\right) > \beta$.
Denote the class of all starlike functions of order β in Δ by $S^{\star}(\beta)$.

Definition 1.2. A function $f \in S$ is said to be convex of order $\beta(0 \leq \beta < 1)$ if and only if $Re\left(\frac{1+zf''(z)}{f'(z)}\right) > \beta$.
Denote the class of all convex functions of order β in Δ by $C(\beta)$.

Definition 1.3. A function $f \in S$ is said to be close-to-convex if there is a convex function $g(z)$ such that $Re\left(\frac{f'(z)}{g'(z)}\right) > 0$.

We note that $f(z)$ is not required to be univalent, and $g(z)$ need not be a function belonging to the class S. It is readily observed that every close-to-convex function is univalent [4]. Merkes and Scott [3] proved an interesting result characterizing starlike hypergeometric functions, and Carlson and Shaffer [5] studied various interesting classes of starlike and convex hypergeometric functions.

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