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Fast approximate method for solving nonlinear system of Fredholm-Volterra integral equations

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Abstract

A numerical method for solving nonlinear system of Fredholm-Volterra Hammerstain integral equations of second kind is presented. This method is based on replacement of the unknown functions by truncated series of well known Chebyshev expansion of functions. The quadrature formula which we use to calculate integral terms can be estimated by Fast Fourier Transform (FFT). Also convergence and rate of convergence are given.

Keywords: Nonlinear system of Fredholm-Volterra integral equation, Chebyshev polynomials, error analysis.
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1 Introduction

In this paper we present a computational method for solving a system of nonlinear Fredholm-Volterra integral equations of Hammerstain type:

$$x_i(s) = y_i(s) + \lambda_1 \sum_{j=1}^{n} \int_0^s K_{ij}(s,t)F(x_j(t))dt + \lambda_2 \sum_{j=1}^{n} \int_0^1 K'_{ij}(s,t)G(x_j(t))dt, \quad i = 1,...,n, \quad 0 \le s,t \le 1. \tag{1}$$

Consider the nonlinear system of integral equation (1). At first we approximate $x_i(t)$ for $i = 1,..,n$, as

$$x_i(t) \simeq \mathbf{C}_i^T \mathbf{T}(t), \tag{2}$$

then we substitute this approximation into eq. (1) to get

$$\mathbf{C}_i^T\mathbf{T}(s) = y_i(s) + \lambda_1 \sum_{j=1}^{n} \int_0^s K_{ij}(s,t)F(\mathbf{C}_j^T\mathbf{T}(t))dt + \lambda_2 \sum_{j=1}^{n} \int_0^1 K'_{ij}(s,t)G(\mathbf{C}_j^T\mathbf{T}(t))dt, \quad i = 1,...,n, \quad 0 \le s,t \le 1. \tag{3}$$

In order to use Gaussian integration formula for eq. (3), we transfer the intervals $[0, s_l]$ and $[0, 1]$ into interval $[-1, 1]$ by transformations

$$\tau_1 = \frac{2}{s_l}t - 1, \quad \tau_2 = 2t - 1.$$

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