Recurrent second fundamental form in submanifolds of Kenmotsu manifolds

Mohammad Bagher Kazemi*
University of Zanjan

Abstract

In this paper, we study recurrent submanifolds of Kenmotsu manifolds. We show that they are totally geodesic. Moreover, generalized recurrent submanifolds of Kenmotsu manifolds are investigated.

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1 Preliminaries

Let $(\tilde{M}, \phi, \xi, \eta, \tilde{g})$ be a $2n+1$ dimensional almost contact manifold, where ϕ, ξ, η and $\tilde{g}$ are $(1,1)$-tensor field, vector field, 1-form and a Riemannian metric respectively, which satisfy the following conditions

$$\phi\xi = 0, \eta(\phi X) = 0, \eta(\xi) = 1,$$

$$\phi^2 X = -X + \eta(X)\xi \ , \ \tilde{g}(\xi, X) = \eta(X),$$

$$(\tilde{\nabla}_X \eta)(Y) = g(X,Y) - \eta(X)\eta(Y), \quad \forall X, Y \in \mathcal{T}\tilde{M}.$$

An almost contact manifold is said to be a Kenmotsu manifold if

$$(\tilde{\nabla}_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X, \tag{1}$$

where $\tilde{\nabla}$ is the Riemannian connection of $\tilde{g}$ [2]. In a Kenmotsu manifold the following relation holds

$$(\tilde{\nabla}_X \xi) = X - \eta(X)\xi. \tag{2}$$

Let (M, g) be a submanifold of a Riemannian manifold $(\tilde{M}, \tilde{g})$. If ∇ be the Levi-Chivita connections of M, then from Gauss and Weingarten formulas we have [5]

$$\tilde{\nabla}_Y X = \nabla_Y X + B(X,Y) \ , \ \tilde{\nabla}_Y V = D_Y V - A_V Y, \tag{3}$$

for any X and Y in $\mathcal{T}M$ and V in $(\mathcal{T}M)^\perp$. In (3), B, A and D are the second fundamental form, associated second fundamental form (shape operator) and normal connection on the $(\mathcal{T}M)^\perp$, respectively.

Let M be a submanifold of an almost contact manifold $(\tilde{M}, \phi, \xi, \eta, \tilde{g})$. M is said to be an invariant submanifold if the vector field ξ is tangent to M and $\phi T_p(M) \subset T_p M$ for all $p \in M$. Also, M is said to be an anti-invariant, if $\phi T_p(M) \subset T_p(M)^\perp$ for all $p \in M$ [4] .

*Speaker