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Continuous and Exact-Continuous Frame for Hilbert Spaces

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Abstract

In this paper we remaind some property for the continuous frame for the Hilbert space and show that the continuous frame and exact-continuous frame are equal under some conditions.

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1 Introduction

Let $\mathcal{H}$ be a complex Hilbert space and $\mathcal{M}$ be a measure space with a positive measure μ. A continuous frame is a family $\{\psi_k\}_{k\in\mathcal{M}}$ for which the following hold:
(c1) For all $h \in \mathcal{H}$, the mapping

$$\Phi : \mathcal{M} \to \mathcal{C}, \Phi(k) =< h, \psi_k >$$

is a measurable function on $\mathcal{M}$.
(c2) There exist constants $A, B > 0$ such that

$$A\|h\|^2 \leq \int_{\mathcal{M}} |< h, \psi_k >|^2 \, d\mu(k) \leq B\|h\|^2, \forall h \in \mathcal{H}$$

If $A = B$ then the continuous frame is called continuous tight frame and if $A = B = 1$ then the continuous frame is called normalized continuous tight frame.

For the sake of simplicity we assume that the mapping $x \longmapsto < f, \psi_k >$ is weakly continuous for all $k \in \mathcal{M}$. Note that if $\mathcal{M}$ be a countable set and μ the counting measure then we obtain the usual definition of a (discrete) frame. By Cauchy-Schwartz inequality

$$\int_{\mathcal{M}} < f, \psi_k >< \psi_k, g > d\mu(k)$$

is well defined for all $f, g \in \mathcal{H}$.

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