46th Annual Iranian Mathematics Conference
25-28 August 2015
Yazd University

A computational algorithm for the inverse of positive definite tri-diagonal matrices

T. Dehghn Niri*
Yazd University

Abstract

In this paper, employing the general Cholesky Q.I.F. factorization, an efficient algorithm is developed to find the inverse of a general positive definite tridiagonal matrix.

Keywords: Cholesky Q.I.F. factorization, Positive definite tridiagonal.
Mathematics Subject Classification [2010]: 13D45, 39B42

1 Introduction

The linear system of equations whose coefficient matrix is of tri-diagonal type of the form

$$T = \begin{bmatrix} a_1 & c_1 & \circ & \cdots & \circ \\ c_1 & a_2 & c_2 & \ddots & \vdots \\ \circ & c_2 & a_3 & \ddots & \circ \\ \vdots & \ddots & \ddots & \ddots & c_{n-1} \\ \circ & \cdots & \circ & c_{n-1} & a_n \end{bmatrix} \tag{1.1}$$

is of special importance in many scientific and engineering applications. For example in parallel computing and in solving differential equations using finite differences.

2 Cholesky Q.I.F. factorization

Consider the linear system $Ax = f$, where A is an $n \times n$ symmetric positive definite matrix. Suppose $n = 2m - 2$. Assume that there exists a matrix W such that , $A = WW^T$,where

$$W = \begin{bmatrix} w_{1,1} & w_{1,2} & & \cdots & \cdots & & & w_{1,n} \\ \circ & w_{2,2} & & \cdots & \cdots & & w_{2,n} & \circ \\ & \circ & \ddots & & & . & \circ & \\ \vdots & & & w_{m-1,m-1} & w_{m-1,m} & \circ & & \\ \vdots & \vdots & \circ & \circ & w_{m,m} & \circ & \vdots & \vdots \\ & \circ & . & & & . & \circ & \\ \circ & \circ & w_{n-1,3} & \cdots & \cdots & & w_{n-1,n-1} & \circ \\ \circ & w_{n,2} & & \cdots & \cdots & \cdots & \cdots & w_{n,n} \end{bmatrix}$$

*Speaker