



On symmetric hypergraphs

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Abstract

A hypergraph Γ is said to be symmetric if its automorphism group acts transitively on the set of its 1-arcs. In this paper we study some properties of the symmetric hypergraphs and then we connect the notions of symmetric hypergraphs with locally symmetric graphs. Besides, given a symmetric hypergraph, we prove that there are infinitely many symmetric hypergraphs that cover the given one.

KeyWords: Symmetric hypergraph, s -arc transitive, locally s -arc transitive.

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1 Introduction

For $s \geq 0$, an s -arc in a graph H is an $(s+1)$ -tuple $(v_0, v_1, \dots, v_s)$ of vertices such that each v_i is adjacent to v_{i+1} while $v_i \neq v_{i+2}$. Let G be a subgroup of $\text{Aut}(H)$. We say that H is (G, s) -arc transitive, or just s -arc transitive, if G acts transitively on the set of s -arcs of H . H is said to be *symmetric* if G acts transitively on the set of 1-arcs of H .

Given $G \leq \text{Aut}(H)$, we say that H is *locally* (G, s) -arc transitive, or just *locally* s -arc transitive, if for each vertex α , the stabilizer G_α acts transitively on the set of s -arcs starting at α . We say that H is *locally symmetric*, if it is locally 1-arc transitive. The study of s -arc transitive graphs and locally s -arc transitive graphs goes back to Tutte [6]. In this paper we consider a natural extension of symmetric graphs to the symmetric hypergraphs.

For $s \geq 0$, an s -arc in a hypergraph $\Gamma = (V, E)$ is an alternate sequence of vertices and edges,

$$(v_0, e_1, v_1, e_2, v_2, \dots, v_{s-1}, e_s, v_s)$$

where each edge e_i is incident to the vertices v_{i-1}, v_i , $1 \leq i \leq s$ and two consecutive vertices or edges are distinct. The hypergraph Γ is s -arc transitive if it has an automorphism group which acts transitively on the set of s -arc. Γ is said to be *symmetric* if its automorphism group acts transitively on the set of 1-arcs.

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