On the cospectrality of graphs

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Abstract

Richard Brualdi proposed in [Research problems from the Aveiro workshop on graph spectra, *Linear Algebra and its Applications*, **423** (2007) 172-181.] the following problem:
(Problem AWGS.4) Let G_n and G'_n be two nonisomorphic graphs on n vertices with spectra

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \quad \text{and} \quad \lambda'_1 \geq \lambda'_2 \geq \cdots \geq \lambda'_n,$$

respectively. Define the distance between the spectra of G_n and G'_n as

$$\lambda(G_n, G'_n) = \sum_{i=1}^{n} (\lambda_i - \lambda'_i)^2 \quad \Big(\text{or use } \sum_{i=1}^{n} |\lambda_i - \lambda'_i|\Big).$$

Define the cospectrality of G_n by

$$cs(G_n) = \min\{\lambda(G_n, G'_n) \ : \ G'_n \ \text{not isomorphic to } G_n\}.$$

Let

$$\mathrm{cs}_n = \max\{\mathrm{cs}(G_n) \ : \ G_n \ \text{a graph on } n \text{ vertices}\}.$$

Problem A. Investigate $\mathrm{cs}(G_n)$ for special classes of graphs.
Problem B. Find a good upper bound on cs_n.
In this paper we study Problem A and determine the cospectrality of all complete bipartite graphs by the Euclidian distance. Let $K_{p,q}$ be the complete bipartite graphs with parts of sizes p and q. We prove that for every positive integers p and q there are some positive integers p', q' and a non-negative integer r such that $\mathrm{cs}(K_{p,q}) = \lambda(K_{p,q}, K_{p',q'} + rK_1)$. As a consequence we determine the cospectrality of stars.

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