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# Derivation of a crack opening deflection relationship for fibre reinforced concrete panels using a stochastic model: Application for predicting the flexural behaviour of round panels using stress crack opening diagrams

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ABSTRACT

This study is aimed at proposing a simple analytical model to investigate the post-cracking behaviour of FRC panels, using an arbitrary tension softening, stress crack opening diagram, as the input. A new relationship that links the crack opening to the panel deflection is proposed. Due to the stochastic nature of material properties, the random fibre distribution, and other uncertainties that are involved in concrete mix, this relationship is developed from the analysis of beams having the same thickness using the Monte Carlo simulation (MCS) technique. The softening diagrams obtained from direct tensile tests are used as the input for the calculation, in a deterministic way, of the mean load displacement response of round panels. A good agreement is found between the model predictions and the experimental results.


## 1. Introduction

The use of fibre reinforced concrete (FRC) as a construction material has continuously grown over the last decades due to its economical advantages in labour and material costs compared to conventional reinforced concrete. Randomly distributed fibres in concrete also contribute to the enhancement of the concrete post-cracking properties and durability when well proportioned and good quality FRC mixes are used. Fibres significantly prevent shrinkage cracking, reduce brittleness due to impact loading, compression or in unreinforced members subjected to shear or tensile forces, control cracking propagation, improve water tightness, etc. For these reasons, FRC has been successfully used in shotcrete, precast concrete, elevated slabs, bridge decks, pavements, industrial floors, seismic resisting structures, repair, etc.

The post-crack behaviour of FRC has been experimentally investigated based on a variety of tests. Although the actual post-cracking response of FRC is better defined by the stress–crack width $\sigma(W)$ relationship measured in a direct tension test, the vast majority of reported test results in the literature involve beams due to the ease of performing bending tests. Flexural beam tests [2], however, have two main disadvantages: the $\sigma(W)$ relationship cannot be obtained directly and tests on small FRC elements engender important scatters in the results that are not representative of the actual in situ conditions [7]. Those observations encouraged many investigators to perform tests on FRC square or round panels of large dimension [1,12,13,17,20]. Despite the better representation of concrete volumes in panel tests, the actual post-cracking behaviour of FRC in tension cannot be determined satisfactory using analytical approaches by any of the proposed tests in the literature unless the softening mechanism is well understood, and the level of deformation and strain-softening characteristics of the material are known.

Contributions in the literature dealing with the post-crack response determination of FRC panels using simple approaches are very limited. Marti et al. [20] developed a simple theoretical approach that accounts for the random fibre distribution for the analysis of slabs. Their model is based a priori on a predefined parabolic softening relationship that considerably simplifies the derivation of load–deflection curves from yield line theory. Recently, Khaloo and Afsari [17] used the same approach for analyzing their tested slabs. Tran et al. [30] proposed an interesting formulation for determining the nonlinear load–deflection response of the round panel [1]. Their formulation uses the yield line theory based on the flexural capacity of beams of similar composition and thickness. They did not use an explicit stress crack–opening relationship; they rather adopted the moment crack rotation angle response of beams as the input.

For the case of FRC beams, Zhang and Stang [32] proposed an analytical formulation that provides, with satisfactory accuracy, the load–displacement response for an arbitrary inputted stress crack-opening diagram. They use an additional relationship derived from fracture mechanics that links the crack mouth opening displacement (CMOD) to the external moment and the crack parameter. For the case

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