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Two modified HS type conjugate gradient methods for unconstrained optimization problems☆

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ABSTRACT

Based on the modified secant equation, we propose two new HS type conjugate gradient formulas. Their forms are similar to the original HS conjugate gradient formula and inherit all nice properties of the HS method. By utilizing the technique of the three-term HS method in Zhang et al. (2007) [15], without the requirement of truncation and convexity of the objective function, we show that one with Wolfe line search and the other with Armijo line search are globally convergent. Moreover, under some mild conditions, the linear convergence rate of the two modified methods is established. The numerical results show that the proposed methods are efficient.

1. Introduction

Let function $f : R^n \to R$ be continuously differentiable. Consider the unconstrained optimization problem

$$\min f(x), \quad x \in R^n. \tag{1.1}$$

We denote by $g(x)$ the gradient of f at x. A conjugate gradient method generates a sequence of iterates by letting

$$x_{k+1} = x_k + \alpha_k d_k \tag{1.2}$$

and the direction d_k is defined by

$$d_{k+1} = \begin{cases} -g_0, & \text{if } k = 0, \\ -g_{k+1} + \beta_k d_k, & \text{if } k \geq 1, \end{cases} \tag{1.3}$$

where $g_k = g(x_k)$, $\alpha_k > 0$ is a steplength obtained by a line search, and β_k is a scalar. Some well-known conjugate gradient methods include the Hestenes–Stiefel (HS) method [1], the Fletcher–Reeves (FR) method [2], the Polak–Ribière–Polyak (PRP) method [3,4], the conjugate descent (CD) method [5], the Liu–Storey (LS) method [6], and the Dai–Yuan (DY) method [7].

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