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Existence of multiple periodic solutions for an SIR model with seasonality[☆]

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ABSTRACT

We study an SIR model with a seasonal contact rate and a staged treatment strategy, which is an extension of our previous work [Z. Bai, Y. Zhou, Existence of two periodic solutions for a non-autonomous SIR epidemic model, Appl. Math. Model. 35 (2011) 382–391]. It is proved that the persistence and extinction of the disease are determined by the basic reproductive number ($\mathcal{R}_0$) and a threshold parameter ($\mathcal{R}_c$). It is obtained that the model exhibits two different bistable behaviors under certain conditions: the stable disease-free state coexists with a stable endemic periodic solution, and three endemic periodic solutions coexist with two of them being stable. Numerical simulations are presented to illustrate theoretical results.

1. Introduction

Many infectious diseases of humans fluctuate over time and often show seasonal patterns of incidence, such as measles, mumps, rubella, chickenpox, diphtheria, pertussis and influenza [1]. Seasonal fluctuations in the transmission of infectious diseases imply that the corresponding mathematical models may admit periodic solutions. Thus, it is interesting and important to study the existence and the number of periodic solutions in epidemiological models.

There have been many studies devoted to explaining the periodicity in disease incidence rates (see [1] for a review). Although different epidemiological mechanisms have been given for the existence of periodic solutions, the periodic contact rate may be a main force driving periodicity. For example, Hethcote [2] obtained asymptotic behavior of periodic solutions through a simple SIS model with periodic infection rate. Schwartz [3] showed that multiple stable periodic solutions can coexist for a fixed set of parameters in a seasonally forced nonlinear SEIR model. Kuznetsov and Piccardi [4] used sinusoidally varying contact rates in SEIR and SIR models, and showed the coexistence of two periodic solutions, namely, flip and tangent bifurcations. For other studies on the dynamical properties of periodic epidemic models, readers can refer to [5–8] and references therein.

Treatment, including isolation or quarantine, is an important way to reduce the new infection of a disease, such as measles, tuberculosis and flu [9–11]. Many studies exploring the role of treatment functions in their dynamic epidemic models presented interesting mathematical phenomena, such as bistability and periodicity [12–15]. Recently, by taking into account seasonality of the disease, Bai and Zhou [16] investigated a periodic SIR model with a constant treatment, and

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