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Time maps and exact multiplicity results for one-dimensional prescribed mean curvature equations. II

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ABSTRACT

We consider non-classical solutions of the quasilinear boundary value problem

$$\begin{cases} -\left(\dfrac{u'}{\sqrt{1+(u')^2}}\right)' = \lambda f(u), \quad x \in (-L, L), \\ u(-L) = u(L) = 0, \end{cases}$$

where λ and L are positive parameters. We give complete descriptions of the structure of bifurcation curves and determine the exact numbers of positive non-classical solutions of the model problems for various nonlinearities $f(u) = e^u$, $f(u) = (1+u)^p (p > 0)$, $f(u) = e^u - 1$, $f(u) = u^p (p > 0)$, and $f(u) = a^u (a > 0)$. The methods used are elementary and based on a detailed analysis of time maps. Moreover, for the case $f(u) = |u|^{p-1}u (p > 0)$, we also obtain the exact number of all sign-changing non-classical solutions and show the global structure of bifurcation curves.

1. Introduction

Consider the following quasilinear boundary value problem

$$\begin{cases} -\left(\dfrac{u'}{\sqrt{1+(u')^2}}\right)' = \lambda f(u), \quad x \in (-L, L), \\ u(-L) = u(L) = 0, \end{cases} \tag{1.1}$$

where λ and L are positive parameters. This paper is a continuation of the paper by Pan and Xing [1], where several exact multiplicity results for positive *classical* solutions and sign-changing *classical* solutions of (1.1) are obtained. In the present paper, we study the exact multiplicity of *non-classical* solutions of (1.1).

Classical solutions of (1.1) have been well investigated (see [1–6]). In [3], Habets and Omari considered the problem (1.1) with power nonlinearity $f(u) = u^p$ $(p > 0)$. By the shooting method, they obtained the exact number of positive solutions. They also discussed various combined effects of concave and convex nonlinearities by an upper and lower solution method. In [4], the first author considered the problem (1.1) with exponential nonlinearity $f(u) = e^u$. He obtained the exact

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