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Bifurcation of limit cycles from a heteroclinic loop with a cusp[☆]

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ABSTRACT

In this article, we study the expansion of the first Melnikov function of a near-Hamiltonian system near a heteroclinic loop with a cusp and a saddle or two cusps, obtaining formulas to compute the first coefficients of the expansion. Then we use the results to study the problem of limit cycle bifurcation for two polynomial systems.

1. Introduction and main results

Consider a C^∞ plane system of the form

$$\dot{x} = H_y + \varepsilon p(x, y, \delta), \qquad \dot{y} = -H_x + \varepsilon q(x, y, \delta) \tag{1.1}$$

where $H(x, y)$, $p(x, y, \delta)$ and $q(x, y, \delta)$ are C^∞ functions in (x, y, δ) with δ a vector parameter varying in a compact set $D \subset R^n$. For $\varepsilon = 0$ (1.1) becomes

$$\dot{x} = H_y, \qquad \dot{y} = -H_x \tag{1.2}$$

which is a Hamiltonian system. Hence, system (1.1) is called a near-Hamiltonian system. Usually we suppose the unperturbed system (1.2) has a family of periodic orbits L_h defined by the equation $H(x, y) = h$. The boundary of the family $\{L_h\}$ can be a center or a homoclinic or a heteroclinic loop. An important topic is to study the number of limit cycles of the perturbed system in a neighborhood of a center, a homoclinic or a heteroclinic loop with either saddles or cusps. In this respect, a Melnikov function of the form

$$M(h, \delta) = \oint_{L_h} q\mathrm{d}x - p\mathrm{d}y \tag{1.3}$$

plays an important role; see [1–3].

Let a boundary of the family $\{L_h\}$ be a closed curve having at most two singular points. Then we have the following possibilities for the curve.

(1) It is a homoclinic loop with one hyperbolic saddle.
(2) It is a homoclinic loop with one cusp.
(3) It is a heteroclinic loop having 2 heteroclinic orbits connecting 2 hyperbolic saddles.

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