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Linearly implicit Liénard systems[☆]

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ABSTRACT

The presence of nonlinearities in the capacitance and the inductance in van der Pol type electrical circuits defines a linearly implicit (or quasilinear) counterpart of the classical Liénard systems. When the reactances remain positive, the existence of a unique attracting periodic solution follows, with minor modifications, as in the classical setting. Novel results are obtained when the values of reactances may vanish at certain points of the state space; these points yield singularities of the model, and the existence of an attracting periodic solution can be characterized in terms of the behavior of certain smooth solutions crossing the singular manifold through so-called I-singularities.

1. Introduction

Consider a series circuit composed of a resistor, an inductor and a capacitor. Assuming that the resistor and the inductor are current controlled by characteristics of the form $v_r = f(i_r)$, $\phi = \varphi(i_l)$ (ϕ being the magnetic flux in the inductor), and that the capacitor is voltage controlled by the charge–voltage relation $q = \psi(v_c)$, the dynamical behavior of this circuit is defined by

$$\phi' = v_c - f(i_l) \tag{1a}$$

$$q' = -i_l \tag{1b}$$

$$\phi = \varphi(i_l) \tag{1c}$$

$$q = \psi(v_c). \tag{1d}$$

We have used the relations $v_c = v_l + f(i_r)$, $i_l = i_r = -i_c$ following from Kirchhoff laws.

If both φ and ψ are C^1 mappings, we may rewrite (1a)–(1b) as

$$L(i_l)i_l' = v_c - f(i_l) \tag{2a}$$

$$C(v_c)v_c' = -i_l, \tag{2b}$$

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