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Asymptotic periodicity for some evolution equations in Banach spaces

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ABSTRACT

This work deals with the existence and uniqueness of pseudo-almost periodic and asymptotically ω-periodic mild solutions to some evolution equations in Banach spaces.

1. Introduction

Pseudo-almost periodic and asymptotically periodic functions have many applications in several problems, for example in the theory of functional-differential equations, integral equations and partial differential equations. From an applied perspective asymptotically periodic systems describe our world more realistically and more accurately than periodic ones. There is much interest in developing the qualitative theory and numerical methods of such systems. Related to this subject we refer the reader to [1–3] and the references therein. The concept of pseudo-almost periodicity was introduced in the literature in the early nineties by Zhang [4–8]. Since then, it has attracted the attention of many researchers (see [9–22]). The notion of weighted pseudo-almost periodicity was introduced by Diagana [23] in 2006 and then studied in [24–27]. To construct those weighted pseudo-almost periodic functions, the main idea consists of enlarging the so-called ergodic component. See the recent paper by Agarwal et al. [25], where they discussed existence and uniqueness of a weighted pseudo-almost periodic (mild) solution to a class of semi-linear fractional differential equations. Furthermore, the authors gave applications to abstract partial evolution (respectively, fractional relaxation–oscillation) equations.

This paper is a natural continuation of the work in [28], which investigates the periodicity of evolution equations. Firstly, we study in this work sufficient conditions for the existence and uniqueness of a weighted pseudo-almost periodic (mild) solution to the following semi-linear integral equations with infinite delay of the form

$$u(t) = \int_{-\infty}^{t} a(t-s)\big[Au(s) + f(s, u(s))\big]\mathrm{d}s, \quad t \in \mathbb{R}, \tag{1.1}$$

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