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Existence, uniqueness and behavior of solutions for a class of nonlinear parabolic problems

Maria Michaela Porzio

Dipartimento di Matematica "Guido Castelnuovo", Sapienza Universitá di Roma, Piazzale A. Moro 2, 00185 Roma, Italy

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ABSTRACT

We prove existence, uniqueness, regularity results and estimates describing the behavior (both for large and small times) of a solution u of some nonlinear parabolic equations of Leray-Lions type including the p-Laplacian. In particular we show how the summability of the initial datum u_0 and the value of p influence the behavior of the solution u, producing ultracontractive or supercontractive estimates or extinction in finite time or different kinds of decay estimates.

1. Introduction and statement of results

Let us consider the following nonlinear problems

$$\begin{cases} u_t - \operatorname{div}(a(x,t,u,\nabla u)) = 0 & \text{in } \Omega_T, \\ u = 0 & \text{on } \Gamma, \\ u(x,0) = u_0(x) & \text{on } \Omega, \end{cases} \tag{1.1}$$

where $\Omega_T = \Omega \times (0,T)$, Ω is an open bounded set of $\mathbb{R}^N$, $N \geq 2$, $T > 0$ and $\Gamma = \partial\Omega \times (0,T)$, with $\partial\Omega$ regular (for example satisfying the property of positive geometric density).

Here the function $a(x,t,s,\xi) : \Omega \times (0,T) \times \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}^N$ is a Caratheodory function[1] satisfying, for a.e. $(x,t) \in \Omega_T$ and for every $s \in \mathbb{R}$, ξ and $\eta \in \mathbb{R}^N$ the following classical Leray-Lions structure conditions

$$\alpha|\xi|^p \leq a(x,t,s,\xi)\xi, \quad \alpha > 0,\ 1 < p < N, \tag{1.2}$$

$$|a(x,t,s,\xi)| \leq \beta[|s|^{p-1} + |\xi|^{p-1} + h(x,t)], \quad \beta > 0, \tag{1.3}$$

$$[a(x,t,s,\xi) - a(x,t,s,\eta)][\xi - \eta] > 0, \quad \xi \neq \eta, \tag{1.4}$$

where $h \in L^{p'}(\Omega_T)$, $\frac{1}{p} + \frac{1}{p'} = 1$.

E-mail address: porzio@mat.uniroma1.it.

[1] That is, it is continuous with respect to (s,ξ) for almost every $(x,t) \in \Omega_T$, and measurable with respect to (x,t) for every $(s,\xi) \in \mathbb{R} \times \mathbb{R}^N$.

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