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Blow up oscillating solutions to some nonlinear fourth order differential equations

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ABSTRACT

We give strong theoretical and numerical evidence that solutions to some nonlinear fourth order ordinary differential equations blow up in finite time with infinitely many wild oscillations. We exhibit an explicit example where this phenomenon occurs. We discuss possible applications to biharmonic partial differential equations and to the suspension bridges model. In particular, we give a possible new explanation of the collapse of bridges.

1. Introduction

In this paper, we study the differential equation

$$w''''(s) + kw''(s) + f(w(s)) = 0 \quad (s \in \mathbb{R}) \tag{1}$$

where $k \in \mathbb{R}$ and f is a locally Lipschitz function.

This equation arises in several contexts. With no hope of being exhaustive, let us mention some models which lead to (1). When k is negative (1) is known as the extended Fisher–Kolmogorov equation, whereas when k is positive it is referred to as the Swift–Hohenberg equation; see [1]. Eq. (1) arises in the dynamic phase-space analogy of a nonlinearly supported elastic strut [2] and serves as a model of pattern formation in many physical, chemical or biological systems; see [3,4] and references therein. It may also be used to investigate localization and spreading of deformation of a strut confined by an elastic foundation [5]. A particularly interesting model concerns traveling waves in a suspension bridge; see [6–8] and Section 3.1. Eq. (1) also arises from a suitable transformation of some biharmonic pde's; see [9,10] and Section 3.2. Last but not least, we mention the important book by Peletier–Troy [1] where one can find many other physical models, a survey of existing results, and further references.

The purpose of the present paper is to contribute to a better understanding of possible finite time blow up for solutions to (1) when the nonlinearity f satisfies

$$f \in \mathrm{Lip}_{\mathrm{loc}}(\mathbb{R}), \quad f(t)\,t > 0 \quad \text{for every } t \in \mathbb{R} \setminus \{0\}. \tag{2}$$

Further assumptions on f will be needed in the following. We are interested in necessary and/or sufficient conditions for local solutions to be global.

The starting point of our study are the following results proved in [10].

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